Name ________________________ ID3 _ _ _

ECS 315: Probability and Random Processes **2018/1**

HW 3 — Due: September 11, 4 PM

Lecturer: Prapun Suksompong, Ph.D.

Instructions

(a) This assignment has 5 pages.

(b) (1 pt) Work and write your answers **directly on these sheets** (not on another blank sheet of paper). Hard-copies are distributed in class.

(c) (1 pt) Write your first name and the last three digits of your student ID in the spaces provided on the upper-right corner of this page.

(d) (8 pt) Try to solve all non-optional problems.

(e) Late submission will be heavily penalized.

Problem 1. If A, B, and C are disjoint events with $P(A) = 0.2$, $P(B) = 0.3$ and $P(C) = 0.4$, determine the following probabilities:

(a) $P(A \cup B \cup C)$

(b) $P(A \cap B \cap C)$

(c) $P(A \cap B)$

(d) $P((A \cup B) \cap C)$

(e) $P(A^c \cap B^c \cap C^c)$

[Montgomery and Runger, 2010, Q2-75]

Problem 2. The sample space of a random experiment is $\{a, b, c, d, e\}$ with probabilities 0.1, 0.1, 0.2, 0.4, and 0.2, respectively. Let A denote the event $\{a, b, c\}$, and let B denote the event $\{c, d, e\}$. Determine the following:

(a) $P(A)$

(b) $P(B)$

(c) $P(A^c)$

(d) $P(A \cup B)$

(e) $P(A \cap B)$

[Montgomery and Runger, 2010, Q2-55]

Problem 3. ***Binomial theorem***: For any positive integer n, we know that

$$(x + y)^n = \sum_{r=0}^{n} \binom{n}{r} x^r y^{n-r}. \tag{3.1}$$

(a) What is the coefficient of $x^{12}y^{13}$ in the expansion of $(x + y)^{25}$?

(b) What is the coefficient of $x^{12}y^{13}$ in the expansion of $(2x - 3y)^{25}$?

(c) Use the binomial theorem (3.2) to evaluate $\sum\limits_{k=0}^{n} (-1)^k \binom{n}{k}$.

Problem 4. Let A and B be events for which $P(A)$, $P(B)$, and $P(A \cup B)$ are known. Express the following probabilities in terms of the three known probabilities above.

(a) $P(A \cap B)$

(b) $P(A \cap B^c)$

(c) $P\left(B \cup (A \cap B^c)\right)$

(d) $P(A^c \cap B^c)$

Extra Questions

Here are some optional questions for those who want more practice.

Problem 5. ***Binomial theorem***: For any positive integer n, we know that

$$(x+y)^n = \sum_{r=0}^{n} \binom{n}{r} x^r y^{n-r}. \tag{3.2}$$

(a) Use the binomial theorem (3.2) to simplify the following sums

(i) $\sum\limits_{\substack{r=0 \\ r \text{ even}}}^{n} \binom{n}{r} x^r (1-x)^{n-r}$

(ii) $\sum\limits_{\substack{r=0 \\ r \text{ odd}}}^{n} \binom{n}{r} x^r (1-x)^{n-r}$

(b) If we differentiate (3.2) with respect to x and then multiply by x, we have

$$\sum_{r=0}^{n} r\binom{n}{r}x^r y^{n-r} = nx(x+y)^{n-1}.$$

Use similar technique to simplify the sum $\sum_{r=0}^{n} r^2\binom{n}{r}x^r y^{n-r}$.

Problem 6.

(a) Suppose that $P(A) = \frac{1}{2}$ and $P(B) = \frac{2}{3}$. Find the range of possible values for $P(A \cap B)$. Hint: Smaller than the interval $[0, 1]$. [Capinski and Zastawniak, 2003, Q4.21]

(b) Suppose that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$. Find the range of possible values for $P(A \cup B)$. Hint: Smaller than the interval $[0, 1]$. [Capinski and Zastawniak, 2003, Q4.22]

Problem 7. (**Classical Probability and Combinatorics**) Suppose n integers are chosen with replacement (that is, the same integer could be chosen repeatedly) at random from $\{1, 2, 3, \ldots, N\}$. Calculate the probability that the chosen numbers arise according to some non-decreasing sequence.