ECS 315: In-Class Exercise # 8

Date: 18 / 09 / 2018			
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Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

(1) Consider events A, B, C, D defined on a sample space Ω.
Suppose

$$P(B) = 1/3,\ P(D) = 1/4,$$

$$P(A|B) = 1/5,\ P(A|B^c) = 3/5,\ P(A|D) = 1.$$

(a) Find $P(A \cap B)$.

$$P(A\cap B) = P(A|B)P(B) = \frac{1}{5}\times\frac{1}{3} = \frac{1}{15}$$

(b) Use the total probability theorem to find P(A).

$$P(A) = P(A|B)P(B) + P(A|B^c)\underbrace{P(B^c)}_{1-P(B) = 1-\frac{1}{3} = \frac{2}{3}} = \frac{1}{5}\times\frac{1}{3} + \frac{3}{5}\times\frac{2}{3} = \frac{1}{15} + \frac{6}{15} = \frac{7}{15}$$

(c) Find P(B|A).

Bayes' theorem (form #1)

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)} = \frac{\frac{1}{5}\times\frac{1}{3}}{\frac{7}{15}} = \frac{1}{7}$$

(d) Find P(A|D^c).

Again, we apply the total probability theorem

Method 1:

$$P(A) = P(A|D)P(D) + P(A|D^c)P(D^c) \qquad P(D^c) = 1-P(D)$$

$$\frac{7}{15} = 1\times\frac{1}{4} + P(A|D^c)\left(1-\frac{1}{4}\right)$$

$$P(A|D^c) = \left(\frac{7}{15}-\frac{1}{4}\right)\times\frac{4}{3} = \frac{13}{\cancel{60}_{15}}\times\frac{\cancel{4}}{3} = \frac{13}{45}$$

Method 2:

$$P(A|D^c) = \frac{P(A\cap D^c)}{P(D^c)} = \frac{13/60}{3/4} = \frac{13}{45}$$

$$P(A\cap D^c) = P(A) - P(A\cap D) = \frac{7}{15}-\frac{1}{4} = \frac{13}{60}$$

$$P(A\cap D) = P(D)P(A|D) = \frac{1}{4}\times 1 = \frac{1}{4}$$

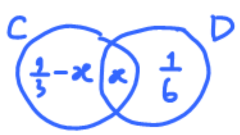
(2) Suppose $P(A)=\frac{1}{2}$ and $P(B)=\frac{1}{3}$. Find $P(A\cap B)$ to make A and B independent.

To make $A \perp\!\!\!\perp B$, we need $P(A\cap B) = P(A)P(B)$

Therefore, $P(A\cap B) = \frac{1}{2}\times\frac{1}{3} = \frac{1}{6}$.

(3) Suppose $P(C)=\frac{1}{3}$ and $P(D\cap C^c)=\frac{1}{6}$. Find $P(C\cap D)$ to make C and D independent.

To make $C \perp\!\!\!\perp D$, we need $P(C\cap D) = P(C)P(D)$. Let $P(C\cap D) = x$.



$$x = \frac{1}{3}\left(x+\frac{1}{6}\right)$$

$$3x = x+\frac{1}{6}$$

$$2x = \frac{1}{6}$$

$$x = \frac{1}{12}$$

Alternatively, we need $P(D\cap C^c) \overset{?}{=} P(D)P(C^c)$ (Another equivalent property)

$$\frac{1}{6} = P(D)\left(1-\frac{1}{3}\right) \Rightarrow P(D) = \frac{1}{6}\times\frac{3}{2} = \frac{1}{4} \Rightarrow P(C\cap D) = P(C)P(D) = \frac{1}{3}\times\frac{1}{4} = \frac{1}{12}$$