ECS 315: In-Class Exercise # 18

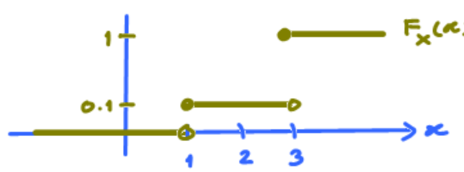
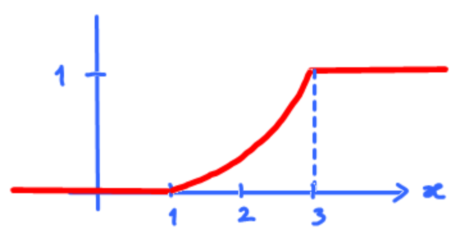
Date: **15** / **11** / 2018	
Name	ID (last 3 digits)
Prapun	**5 5 5**

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups after the midterm.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

1. In this question, we consider two distributions for a random variable X. In part (a), which corresponds to the second column in the table below, X is a ***discrete*** random variable with its pmf specified in the first row. In part (b), which corresponds to the third column, X is a ***continuous*** random variable with its pdf specified in the first row.

$p_X(1) = 0.1 \quad p_X(3) = 0.9$

	$p_X(x) = \begin{cases} \frac{1}{10}x^2, & x \in \{1,3\}, \\ 0, & \text{otherwise.} \end{cases}$	$f_X(x) = \begin{cases} \frac{3}{26}x^2, & x \in (1,3), \\ 0, & \text{otherwise.} \end{cases}$
Find the cdf $F_X(x)$	For discrete RV, it may be easier to work on the plot of cdf first, then come back here. $F_X(x) = \begin{cases} 0, & x < 1, \\ 0.1, & 1 \le x < 3, \\ 1, & x \ge 3. \end{cases}$	For $x \in (1,3)$, $F_X(x) = \int_1^x f_X(t)\,dt = \int_1^x \frac{3}{26}t^2\,dt = \frac{3}{26}\frac{t^3}{3}\Big\vert_1^x$ $= \frac{1}{26}(x^3-1)$ $F_X(x) = \begin{cases} 0, & x \le 1, \\ \frac{1}{26}(x^3-1), & 1 < x < 3, \\ 1, & \text{otherwise} \end{cases}$
Plot the cdf $F_X(x)$		
Find $\mathbb{E}X$	$\mathbb{E}X = \sum_x x\,p_X(x)$ $= 1\times0.1 + 3\times0.9$ $= 0.1 + 2.7$ $= 2.8$	$\mathbb{E}X = \int_{-\infty}^{\infty} x f_X(x)\,dx = \int_1^3 x\frac{3}{26}x^2\,dx$ $= \frac{3}{26}\frac{x^4}{4}\Big\vert_1^3 = \frac{3}{4\times26}(3^4-1^4)$ $= \frac{3}{\cancel{4}_{1}\times\cancel{26}_{13}}\times\cancel{\cancel{80}}^{\,10} = \frac{30}{13} \approx 2.3077$
Find $\mathbb{E}[X^2]$	$\mathbb{E}[X^2] = \sum_x x^2 p_X(x)$ $= 1^2\times0.1 + 3^2\times0.9$ $= 0.1 + 8.1$ $= 8.2$	$\mathbb{E}[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx = \int_1^3 x^2\frac{3}{26}x^2\,dx$ $= \frac{3}{26}\frac{x^5}{5}\Big\vert_1^3 = \frac{3}{26\times5}(3^5-1^5)$ $= \frac{3}{\cancel{26}_{13}\times5}\times\cancel{242}^{121} = \frac{363}{65} \approx 5.58$