ECS 315: In-Class Exercise # 17

Date: **13** / **11** / 2018	
Name	ID (last 3 digits)
Prapun	**5 5 5**

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups after the midterm.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

In this question, we consider two distributions for a random variable X. In part (a), which corresponds to the second column in the table below, X is a ***discrete*** random variable with its pmf specified in the first row. In part (b), which corresponds to the third column, X is a ***continuous*** random variable with its pdf specified in the first row.

	$p_X(x)=\begin{cases} cx^2, & x\in\{1,3\},\\ 0, & \text{otherwise.}\end{cases}$	$f_X(x)=\begin{cases} cx^2, & x\in(1,3),\\ 0, & \text{otherwise.}\end{cases}$
Find c	"$\sum = 1$" : $p_X(1)+p_X(3)=1$ $c1^2+c3^2=1$ $10c=1$ $c=\frac{1}{10}$	$\int_{-\infty}^{\infty} f_X(x)\,dx = \int_1^3 cx^2\,dx = c\frac{x^3}{3}\Big\rvert_1^3$ $= 9c-\frac{c}{3} = c\frac{26}{3}$ "$\int = 1$" : $c\frac{26}{3}=1 \Rightarrow c=\frac{3}{26}\approx 0.1154$
Find $P[X=1]$	$P[X=1]=p_X(1)=cx^2\Big\rvert_{x=1}=c=\frac{1}{10}$	Method 1: X is a continuous RV. $P[X=x]=0$ for any x. Therefore, $P[X=1]=0$. Method 2: X is a continuous RV. $P[X=1]=\int_1^1 f_X(x)\,dx = \int_1^1 cx^2\,dx = c\frac{x^3}{3}\Big\rvert_1^1$ $=0$
Find $P[1<X<2]$	The possible values of this RV are 1 and 3. So, there is no possible value in the open interval $(1,2)$. Therefore, $P[1<X<2]=0$	$P[1<X<2]=\int_1^2 f_X(x)\,dx = \int_1^2 cx^2\,dx$ $= c\frac{x^3}{3}\Big\rvert_1^2 = \frac{c}{3}(2^3-1^3) = \frac{7c}{3}$ $=\frac{7}{3}\times\frac{3}{26}=\frac{7}{26}\approx 0.2692$
Find $P[X>2]$	Again, the possible values of this RV are 1 and 3. Only "3" satisfies the condition ">2". Therefore $P[X>2]=p_X(3)=cx^2\Big\rvert_{x=3}=9c=\frac{9}{10}$	Method 1: $P[X>2]=\int_2^\infty f_X(x)\,dx = \int_2^3 cx^2\,dx$ $= c\frac{x^3}{3}\Big\rvert_2^3 = \frac{c}{3}(3^3-2^3) = c\frac{19}{3}$ $=\frac{3}{26}\times\frac{19}{3}=\frac{19}{26}\approx 0.7308$ Method 2: Consider the following partition of Ω: $\Omega=[X\le 1]\cup[1<X<2]\cup[X=2]\cup[X>2]$ By finite additivity, we have $P(\Omega)=P[X\le1]+P[1<X<2]+P[X=2]+P[X>2]$ $1 = 0+\frac{7}{26}+0+P[X>2]$ $\Rightarrow P[X>2]=1-\frac{7}{26}=\frac{19}{26}$