

# ECS 315: In-Class Exercise # 16

## Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups after the midterm.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: **06 / 11** / 2018

Name

ID (last 3 digits)

**Prapun**

**5 5 5**

Find  $\mathbb{E}[X^2]$ ,  $\mathbb{E}[(X+1)^2]$ , and  $\text{Var}[X]$  of the random variable  $X$  defined below:

<table style="margin-left: auto; margin-right: auto;"> <tr> <th style="border: none;"><math>x</math></th> <th style="border: none;"><math>p_X(x)</math></th> </tr> <tr> <td style="border: none;">-1</td> <td style="border: none;"><math>\frac{-1+2}{8} = \frac{1}{8}</math></td> </tr> <tr> <td style="border: none;">1</td> <td style="border: none;"><math>\frac{1+2}{8} = \frac{3}{8}</math></td> </tr> <tr> <td style="border: none;">2</td> <td style="border: none;"><math>\frac{2+2}{8} = \frac{4}{8}</math></td> </tr> </table>	$x$	$p_X(x)$	-1	$\frac{-1+2}{8} = \frac{1}{8}$	1	$\frac{1+2}{8} = \frac{3}{8}$	2	$\frac{2+2}{8} = \frac{4}{8}$	$p_X(x) = \begin{cases} \frac{x+2}{8}, & x \in \{-1, 1, 2\}, \\ 0, & \text{otherwise.} \end{cases}$	$p_X(x) = \begin{cases} 0.25, & x = 1, 3, \\ 0.5, & x = 2, \\ 0, & \text{otherwise.} \end{cases}$
$x$	$p_X(x)$									
-1	$\frac{-1+2}{8} = \frac{1}{8}$									
1	$\frac{1+2}{8} = \frac{3}{8}$									
2	$\frac{2+2}{8} = \frac{4}{8}$									
$\mathbb{E}[X]$	1.25	2								
$\mathbb{E}[X^2]$ $\equiv \sum_x x^2 p_X(x)$	$= (-1)^2 \times \frac{1}{8} + 1^2 \times \frac{3}{8} + 2^2 \times \frac{4}{8}$ $= \frac{1}{8} (1 + 3 + 16) = \frac{20}{8} = \frac{5}{2} = 2.5$	$= 1^2 \times \frac{1}{4} + 2^2 \times \frac{1}{2} + 3^2 \times \frac{1}{4}$ $= \frac{1}{4} + 2 + \frac{9}{4} = 2 + \frac{10}{4} = 2 + \frac{5}{2} = 4.5$								
$\mathbb{E}[(X+1)^2]$ $\equiv \sum_x (x+1)^2 p_X(x)$	$= (-1+1)^2 \times \frac{1}{8} + (1+1)^2 \times \frac{3}{8} + (2+1)^2 \times \frac{4}{8}$ $= 0 + \frac{4 \times 3 + 9 \times 4}{8} = \frac{12}{2} = 6$ <p>Alternatively,</p> $\mathbb{E}[(X+1)^2] = \mathbb{E}[X^2 + 2X + 1]$ $= \mathbb{E}[X^2] + 2\mathbb{E}[X] + 1 = 2.5 + 2 \times \frac{5}{4} + 1 = 6$	$= (1+1)^2 \times \frac{1}{4} + (2+1)^2 \times \frac{1}{2} + (3+1)^2 \times \frac{1}{4}$ $= \frac{4}{4} + \frac{9}{2} + \frac{16}{4} = 1 + 4.5 + 4 = 9.5$ <p>Alternatively,</p> $\mathbb{E}[(X+1)^2] = \mathbb{E}[X^2] + 2\mathbb{E}[X] + 1$ $= 4.5 + 2 \times 2 + 1 = 9.5$								
$\text{Var}[X]$	$= \mathbb{E}[X^2] - (\mathbb{E}[X])^2 = \frac{5}{2} - \left(\frac{5}{4}\right)^2$ $= \frac{5}{2} - \frac{25}{16} = \frac{40 - 25}{16} = \frac{15}{16} = 0.9375$	$= \mathbb{E}[X^2] - (\mathbb{E}[X])^2 = 4.5 - 2^2$ $= 4.5 - 4 = 0.5$								