ECS 315: In-Class Exercise # 15

Date: 01 / 11 / 2018			
Name	ID (last 3 digits)		
Prapun	5	5	5

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups after the midterm.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

1. You are given an unfair coin with probability of obtaining a heads equal to 2×10^{-17} (p). You toss this coin 2.5×10^{16} (n) times. Use **Poisson approximation** to find the probability that you get “tails for all the tosses”.

Let N be this RV

The number of successes (Hs) in n Bernoulli trials is binomial(n,p) where p is the success probability for each trial.

Here we want to find $P[N=0]$

When n is large and p is small, the binomial RV can be approximated by a Poisson(α) RV where $\alpha = np$

2.5×10^{16} is large; 2×10^{-17} is small

Here, $\alpha = np = 2.5\times10^{16} \times 2\times10^{-17} = 5\times10^{-1} = 0.5 \Rightarrow P[N=0] \approx e^{-\alpha}\frac{\alpha^0}{0!} = e^{-\alpha} = e^{-0.5} \approx 0.6065$

2. Find the expected value of the random variable X defined in each part below:

a. $$p_X(x) = \begin{cases} \frac{x+2}{8}, & x \in \{-1,1,2\}, \\ 0, & \text{otherwise.} \end{cases}$$

x	$p_X(x)$
-1	$\frac{-1+2}{8} = \frac{1}{8}$
1	$\frac{1+2}{8} = \frac{3}{8}$
2	$\frac{2+2}{8} = \frac{4}{8}$

Check: $\frac{1}{8}+\frac{3}{8}+\frac{4}{8} = 1$ ✓

$\mathbb{E}X = \sum_x x\,p_X(x) = (-1)\frac{1}{8} + (1)\frac{3}{8} + (2)\frac{4}{8} = \frac{10}{8} = \frac{5}{4} = 1.25$

b. $$p_X(x) = \begin{cases} 0.25, & x = 1,3, \\ c, & x = 2, \\ 0, & \text{otherwise.} \end{cases}$$

x	$p_X(x)$
1	0.25
2	$c = 0.5$
3	0.25

“$\Sigma = 1$”

$0.25 + c + 0.25 = c + 0.5 = 1$

$c = 0.5$

$\mathbb{E}X = \sum_x x\,p_X(x) = (1)0.25 + (2)0.5 + (3)0.25 = 0.25 + 1 + 0.75 = 2$

c. $$F_X(x) = \begin{cases} 0, & x < 0, \\ 0.3, & 0 \le x < 2, \\ 1, & x \ge 2 \end{cases}$$

This cdf has two jumps; one is @ $x=0$ and another one is @ $x=1$.
The jump sizes are 0.3 and $1-0.3=0.7$, respectively.

$$\Rightarrow p_X(x) = \begin{cases} 0.3, & x=0, \\ 0.7, & x=2, \\ 0, & \text{otherwise.} \end{cases}$$

$\mathbb{E}X = \sum_x x\,p_X(x) = (0)0.3 + (2)0.7 = 1.4$