Error in Computational Tools

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```
>> 1+10^(-14)
ans =
    1.0000
>> 1+10^(-15)
ans =
    1.0000
>> 1+10^(-16)
ans =
     1
```

Error in Computational Tools

- Mathematically, we know that

$$10^{-16} = 1 + 10^{-16} - 1$$

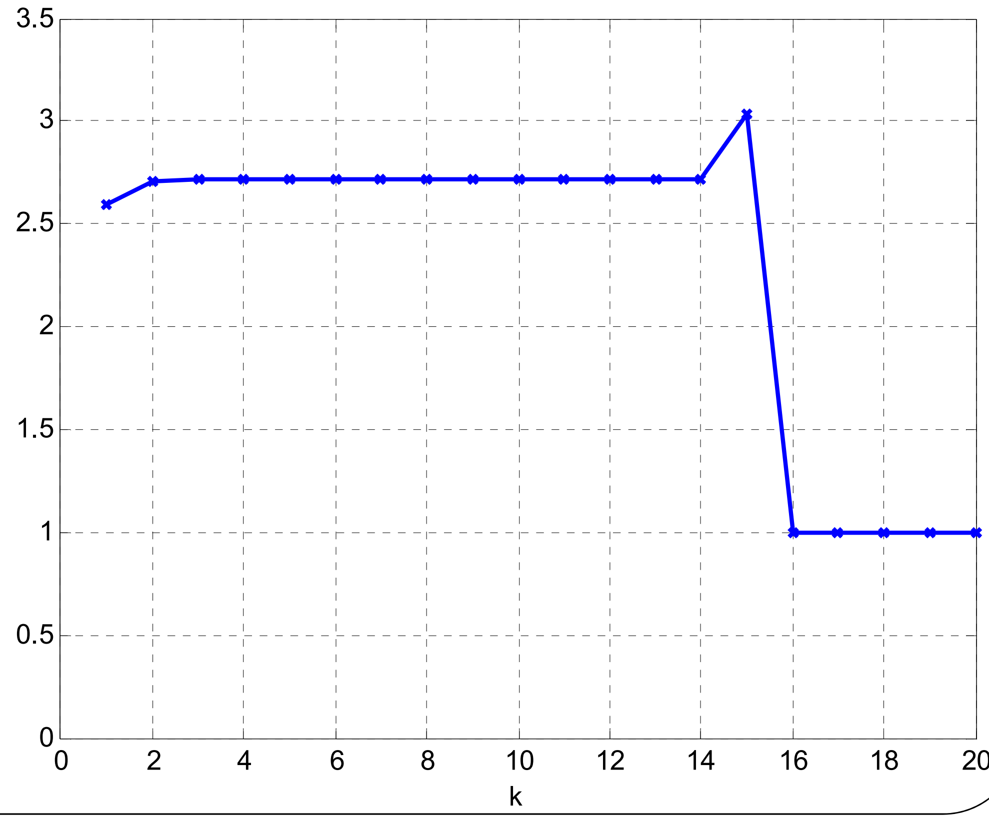
```
>> log10(10^(-16))
ans =
   -16
>> log10(1+10^(-16)-1)
ans =
  -Inf
```

Error in Computational Tools

- Now, let's work on expression of the form

$$\left(1+\frac{1}{10^k}\right)^{10^k}$$

```
>> k = 14; (1+10^(-k))^(10^k)
ans =
   2.716110034087023
>> k = 15; (1+10^(-k))^(10^k)
ans =
   3.035035206549262
>> k = 16; (1+10^(-k))^(10^k)
ans =
     1
```



Error in Computational Tools

- Now, let's work on expression of the form

Theoretically, when k is large, we know that this expression should converge to $e^1 = e \approx 2.7183$.

$$\left(1 + \frac{1}{10^k}\right)^{10^k}$$

```
>> k = 14; (1+10^(-k))^(10^k)
ans =
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>> k = 15; (1+10^(-k))^(10^k)
ans =
   3.035035206549262
>> k = 16; (1+10^(-k))^(10^k)
ans =
     1
```
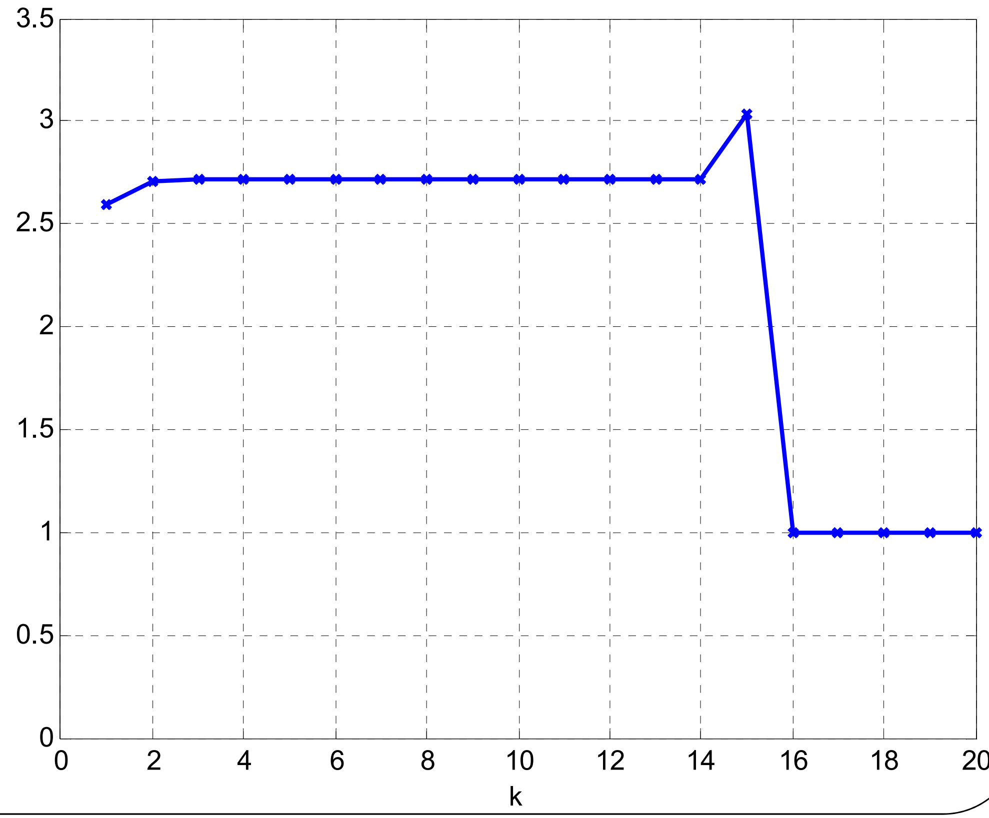
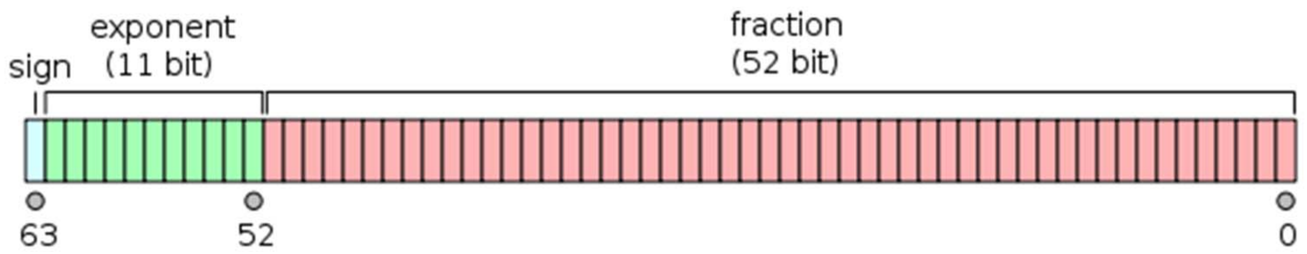



Accuracy of Floating-Point Data

- Computers only represent numbers to a finite precision.
- Computations sometimes yield mathematically nonintuitive results.
- Almost all operations in MATLAB are performed in double-precision arithmetic conforming to the IEEE standard 754.



- The real value assumed by a given 64-bit double-precision datum with a given biased exponent *e* and a 52-bit fraction is

$$(-1)^{\text{sign}}(1.b_{51}b_{50}\ldots b_0)_2 \times 2^{e-1023}$$

Double-Precision Accuracy

- Because there are only a finite number of double-precision numbers, you cannot represent all numbers in double-precision storage.
- On any computer, there is a small gap between each double-precision number and the next larger double-precision number.
- You can determine the size of this gap, which limits the precision of your results, using the **eps** function.
  - This **machine epsilon** gives an upper bound on the relative error due to rounding in floating point arithmetic.
- For example, to find the distance between 1 and the next larger double-precision number, enter

```
>> format long
>> eps(1)
ans =
     2.220446049250313e-16
```

Probability Calculation for Binomial RV

- For binomial RV, probability are of the form
$$\binom{n}{x} p^x (1-p)^{n-x}.$$
- For example, when $x = 0$, we have
$$\binom{n}{0} p^0 (1-p)^{n-0} = (1-p)^n.$$
- When is p small, the number $1 - p$ may not be accurately represented.

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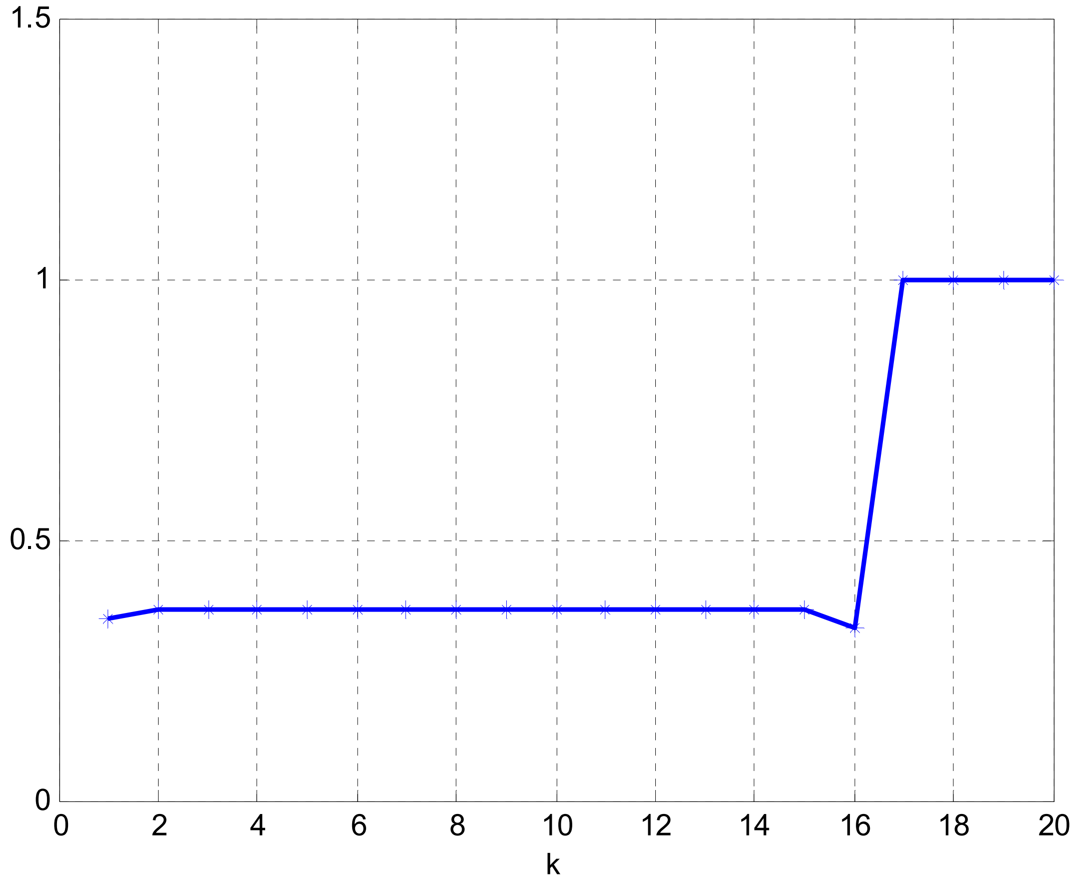
- For example, when $x = 0$, we have
$$\binom{n}{0} p^0 (1-p)^{n-0} = (1-p)^n.$$
- When is p small, the number $1 - p$ may not be accurately represented.
- Consider $p = \frac{1}{10^k}$ and $n = 10^k$.

Probability Calculation for Binomial RV

- $(1-p)^n$ when $p = \frac{1}{10^k}$ and $n = 10^k$.



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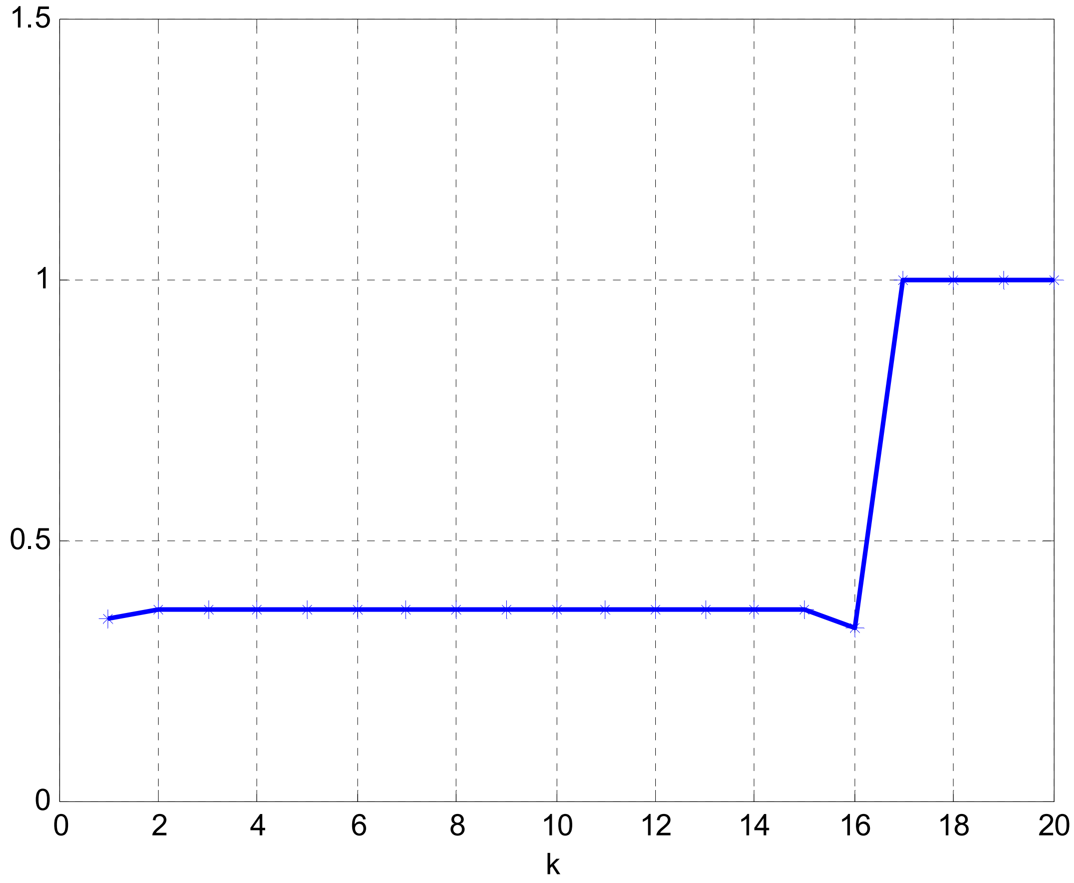

Theoretically, when k is large, we know that this expression should converge to $e^{-1} \approx 0.3679$.