

Probability and Random Processes

ECS 315

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II. Events-Based Probability Theory



Office Hours:

BKD, 6th floor of Sirindhralai building

Wednesday 14:30-15:30

Friday 14:30-15:30

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5 Foundation of Probability Theory



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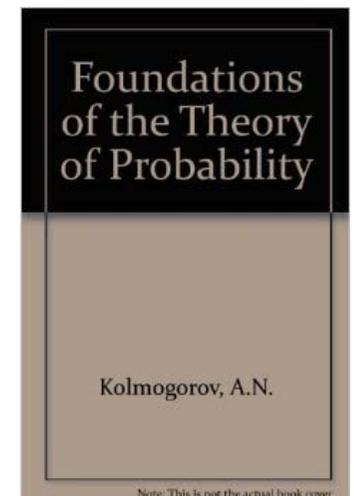
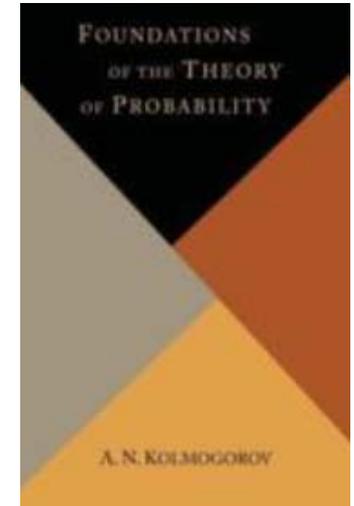
Friday 14:30-15:30

Kolmogorov

- Andrey Nikolaevich Kolmogorov
- Soviet Russian mathematician
- Advanced various scientific fields
 - **probability theory**
 - topology
 - classical mechanics
 - computational complexity.
- 1922: Constructed a **Fourier series** that diverges almost everywhere, gaining international recognition.
- **1933**: Published the book, **Foundations of the Theory of Probability**, laying the modern axiomatic foundations of probability theory and establishing his reputation as the world's leading living expert in this field.

This book is available at

[<https://archive.org/details/foundationsofthe00kolm>]



I learned probability theory from



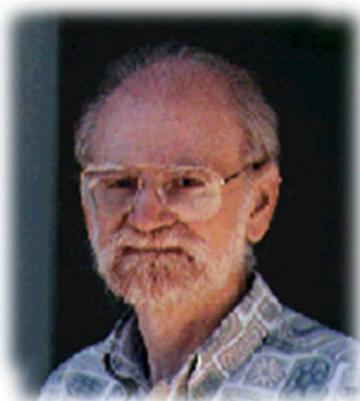
Eugene Dynkin



Philip Protter



Gennady Samorodnitsky



Terrence Fine



Xing Guo



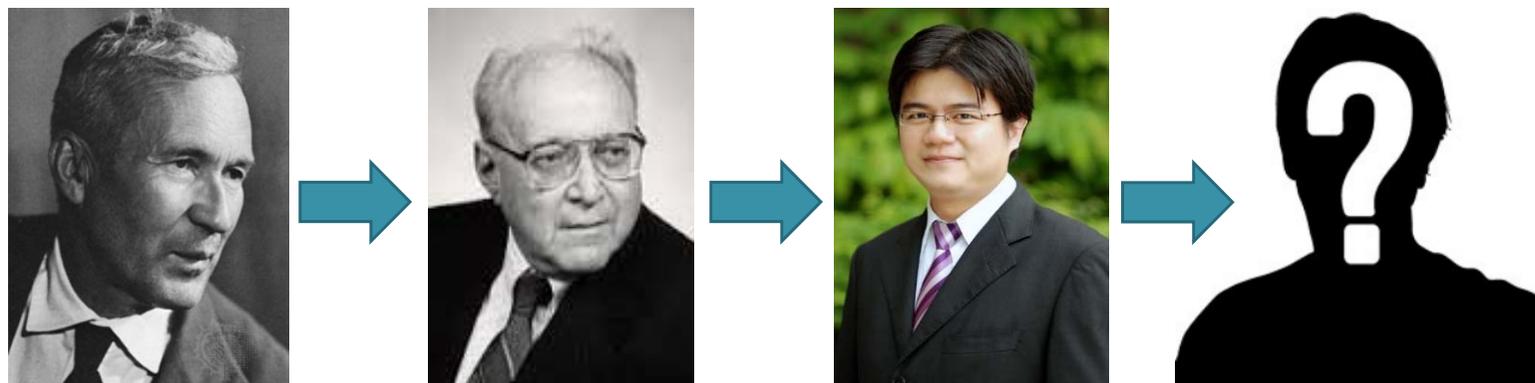
Toby Berger



Rick Durrett



Not too far from Kolmogorov



You can be

the 4th-generation

probability theorists



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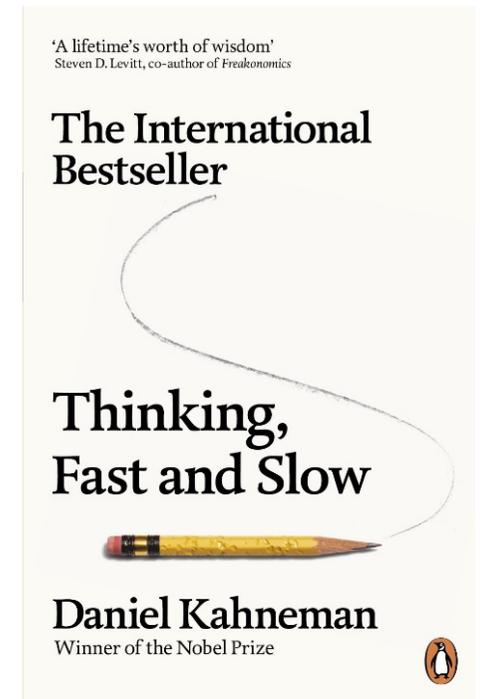
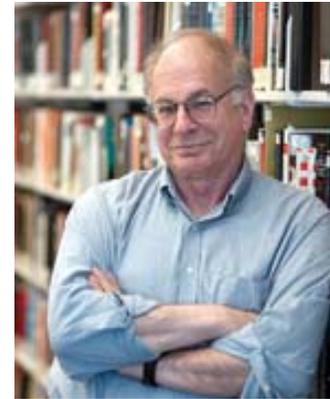
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Event-Based Properties

Daniel Kahneman

- Daniel Kahneman
- Israeli-American **psychologist**
- 2002 **Nobel** laureate 
 - In **Economics**
- Hebrew University, Jerusalem, Israel. 
- Professor emeritus of psychology and public affairs at **Princeton** University's Woodrow Wilson School.
- With Amos **Tversky**, Kahneman studied and clarified the kinds of **misperceptions of randomness** that fuel many of the common fallacies.



K&T: Q1

Imagine a **woman** named **Linda**, **31** years old,
[outspoken = given to expressing yourself freely or insistently]
single, outspoken, and very **bright**. In college
she majored in **philosophy**. While a student she was
deeply concerned with **discrimination** and
social justice and participated in **antinuclear**
[protest]
demonstrations.



- K&T presented this description to a group of 88 subjects and asked them to **rank** the eight statements (shown on the next slide) on a scale of 1 to 8 according to their probability, with 1 representing the **most probable** and 8 representing the **least probable**.



K&T: Q1 - Results

- Here are the results - from **most to least probable**

<i>Statement</i>	<i>Average Probability Rank</i>
Linda is active in the feminist movement.	2.1
Linda is a psychiatric social worker.	3.1
Linda works in a bookstore and takes yoga classes.	3.3
Linda is a bank teller and is active in the feminist movement.	4.1
Linda is a teacher in an elementary school.	5.2
Linda is a member of the League of Women Voters.	5.4
Linda is a bank teller.	6.2
Linda is an insurance salesperson.	6.4



K&T: Q1 – Results (2)

- At first glance there may appear to be nothing unusual in these results: the description was in fact designed to be
 - representative of an active feminist and
 - unrepresentative of a bank teller or an insurance salesperson.

	<i>Statement</i>	<i>Average Probability Rank</i>	
Most probable ↓ Least likely	Linda is active in the feminist movement.	2.1	←
	Linda is a psychiatric social worker.	3.1	
	Linda works in a bookstore and takes yoga classes.	3.3	
	Linda is a bank teller and is active in the feminist movement.	4.1	←
	Linda is a teacher in an elementary school.	5.2	
	Linda is a member of the League of Women Voters.	5.4	
	Linda is a bank teller.	6.2	←
	Linda is an insurance salesperson.	6.4	



K&T: Q1 – Results (3)

- Let's focus on just three of the possibilities and their average ranks.
- This is the order in which **85 percent** of the respondents ranked the three possibilities:

<i>Statement</i>	<i>Average Probability Rank</i>
Linda is active in the feminist movement.	2.1
Linda is a bank teller and is active in the feminist movement.	4.1
Linda is a bank teller.	6.2

- If nothing about this looks strange, then K&T have fooled you



K&T: Q1 - Contradiction

The probability that two events will both occur can never be greater than the probability that each will occur individually!

<i>Statement</i>	<i>Average Probability Rank</i>
Linda is active in the feminist movement.	2.1
Linda is a bank teller and is active in the feminist movement.	4.1
Linda is a bank teller.	6.2



K&T: Q2

- K&T were not surprised by the result because they had given their subjects a **large number of** possibilities, and the connections among the three scenarios could easily have gotten lost in the shuffle.
(eight)
- So they presented the description of Linda to another group, but this time they presented **only three possibilities**:
 - Linda is active in the feminist movement.
 - Linda is a bank teller and is active in the feminist movement.
 - Linda is a bank teller.
- Is it now obvious that the middle one is the least likely?



K&T: Q2 - Results

- To their surprise, **87 percent** of the subjects in this trial also **incorrectly** ranked the probability that “Linda is a bank teller and is active in the feminist movement” higher than the probability that “Linda is a bank teller”.
- If the **details** we are given **fit our mental picture** of something, then the more details in a scenario, the more real it seems and hence the **more probable** we consider it to be
 - even though any act of adding less-than-certain details to a conjecture makes the conjecture less probable.
- Even **highly trained doctors** make this error when analyzing symptoms.
 - 91 percent of the doctors fall prey to the same bias.

[Amos Tversky and Daniel Kahneman, “Extensional versus Intuitive Reasoning: The Conjunction Fallacy in Probability Judgment,” *Psychological Review* 90, no. 4 (October 1983): 293–315.]



Misuse of probability in law

- It is not uncommon for experts in **DNA analysis** to testify at a criminal trial that a DNA sample taken from a crime scene matches that taken from a suspect.
- How certain are such matches?
- When DNA evidence was first introduced, a number of experts testified that **false positives** are **impossible** in DNA testing.
- Today DNA experts regularly testify that the odds of a random person's matching the crime sample are less than **1 in 1 million** or **1 in 1 billion**.
- In Oklahoma a court sentenced a man named Timothy Durham to prison even though **eleven witnesses** had placed him in another state at the time of the crime.



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- Today DNA experts regularly testify that the odds of a random person's matching the crime sample are less than **1 in 1 million** or **1 in 1 billion**.
- In Oklahoma a court sentenced a man named Timothy Durham to **more than 3,100 years in** prison even though **eleven witnesses** had placed him in another state at the time of the crime.



Lab Error

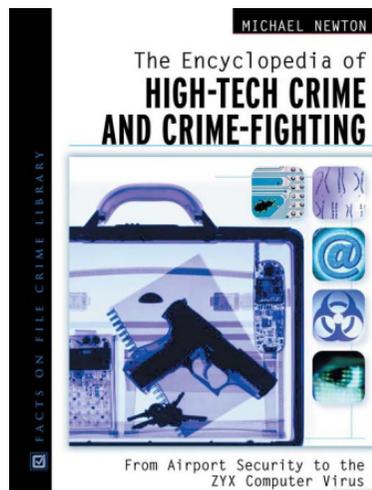
(Human and Technical Errors)

- There is **another statistic** that is often **not presented** to the jury, one having to do with the fact that **labs make errors**, for instance, in **collecting** or **handling** a sample, by accidentally **mixing** or **swapping** samples, or by **misinterpreting** or incorrectly reporting results.
- Each of these errors is rare but not nearly as rare as a random match.
- The Philadelphia City Crime Laboratory admitted that it had swapped the reference sample of the defendant and the victim in a rape case
- A testing firm called Cellmark Diagnostics admitted a similar error.



Timothy Durham's case

- It turned out that in the initial analysis the lab had **failed to** completely **separate the DNA** of the rapist and that of the victim in the fluid they tested, and the combination of the victim's and the rapist's DNA produced a positive result when compared with Durham's.
- A later **retest** turned up the error, and Durham was released after spending nearly **four years** in prison.



DNA-Match Error + Lab Error

- Estimates of the error rate due to human causes vary, but many experts put it at around 1 percent.
- Most jurors assume that given the two types of error—the **1 in 1 billion** accidental match and the **1 in 100 lab-error match**—the overall error rate must be somewhere in between, say 1 in 500 million, which is still, for most jurors, **beyond a reasonable doubt**.

