ECS 303: Quiz 5 Solution

Note that I try to plug in the numbers only at the very last step.

Semester/Year: 2/2009

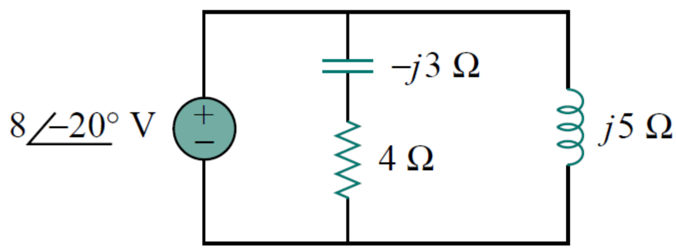
Course Title: *Basic Electrical Engineering*

Name	ID

Instructions

1. Separate into groups of no more than three persons.
2. Closed book. Closed notes.
3. Only one submission is needed for each group. Late submission will not be accepted.
4. **Do not panic.**

1. For the circuit below, find the average power absorbed by each element.



Note: The basic formula to find the average power absorbed by any element is

$$P = \frac{1}{2}\mathrm{Re}\{\vec{V}\vec{I}^*\}.$$

There are two things that you need to be careful with.

1) The $\vec{V}$ must be the voltage across that element and the $\vec{I}$ must be the current through that element.

2) The $\vec{V}$ and $\vec{I}$ should agree with the passive sign convention. (Otherwise, there will be an extra minus sign.)

① At the voltage source:

$\vec{V} = 8\angle -20^\circ$ is already given

we need to find $\vec{I}$ through it.

If we define $\vec{I}$ to go up, then it does not follow the passive sign convention and hence

$$P = -\frac{1}{2}\mathrm{Re}\{\vec{V}\vec{I}^*\}$$

Now, to find $\vec{I}$, we use

$$\vec{I} = \frac{\vec{V}}{\vec{Z}} \quad \text{(Ohm's law)}$$

where $\vec{Z}$ is the total impedance which is

$$= (4 + (-3j)) /\!/ 5j$$

$$= \frac{(4-3j)5j}{4+2j} = \frac{15+20j}{4+2j}$$

Therefore,

$$p = -\frac{1}{2} Re\left\{\vec{V} \times \left(\frac{\vec{V}}{Z}\right)^*\right\}$$

$$= -\frac{1}{2} Re\left\{\frac{|\vec{V}|^2}{Z^*}\right\}$$

$$= -\frac{1}{2} |\vec{V}|^2 Re\left\{\frac{1}{Z^*}\right\}$$

($|\vec{V}| = 8$)

$$Re\left\{\frac{1}{Z^*}\right\} = Re\left\{\frac{4-2j}{15-20j}\right\}$$

$$= Re\left\{\frac{4}{25} + \frac{2}{25}j\right\}$$

$$= \frac{4}{25}$$

$$= -\frac{1}{2} \times 8^2 \times \frac{4}{25} = -5.12\ W$$

So,

$$P_{source} = -5.12\ W$$

(2) We have seen in class that

$$P_{capacitor} = P_{inductor} = 0$$

(3) We know that

a) the source gives 5.12 W

and

b) capacitor and inductor absorb 0 W on average.

Therefore, all the power from the source must be transfer to the resistor.

$$P_{resistor} = -P_{source} = 5.12\ W$$

Remark: If there is more than one resistors, then this power is distributed among the resistors.

Check

For inductor,

$$p = \frac{1}{2} Re\left\{\vec{V}_L \times I_L^*\right\}$$

($\vec{V}_L$: voltage across the inductor; I_L^*: current through the inductor)

$$= \frac{1}{2} Re\left\{I_L \times j\omega L \, I_L^*\right\}$$

(Ohm's law; $j\omega L$: impedance of inductor)

$$= \frac{1}{2} |I_L|^2 Re\{j\omega L\}$$

$$= \frac{1}{2} |I_L|^2 \times 0$$

$$= 0.$$

For capacitor,

$$p = \frac{1}{2} Re\left\{\vec{V}_c \times \vec{I}_c^*\right\}$$

($\vec{V}_c$: Voltage across the capacitor; $\vec{I}_c^*$: current through the capacitor)

$$= \frac{1}{2} Re\left\{\vec{I}_c \times \frac{1}{j\omega C} \times \vec{I}_c^*\right\}$$

(Ohm's law; $\frac{1}{j\omega C}$: impedance of capacitor)

$$= \frac{1}{2} |\vec{I}_c|^2 Re\left\{\frac{1}{j\omega C}\right\}$$

$$= \frac{1}{2} |\vec{I}_c|^2 \times 0$$

$$= 0$$

For resistor,

$$\vec{V}_R = \vec{V} \times \frac{4}{4-3j} \quad \text{by voltage divider.}$$

$$|\vec{V}_R| = |\vec{V}| \times \frac{4}{\sqrt{4^2+3^2}}$$

$$P_R = \frac{1}{2} Re\left\{\vec{V}_R \times \vec{I}_R^*\right\}$$

$$= \frac{1}{2} Re\left\{\vec{V}_R \times \frac{\vec{V}_R^*}{R^*}\right\}$$

(Ohm's law)

$$= \frac{1}{2} |\vec{V}_R|^2 \times Re\left\{\frac{1}{R}\right\} = \frac{1}{2} |\vec{V}_R|^2 \times \frac{1}{R}$$

$$= \frac{1}{2} \times 8^2 \times \frac{4^2}{4^2+3^2} \times \frac{1}{4} = 5.12\ W$$