ECS 303: Quiz 4 Solution

Semester/Year: 2/2009

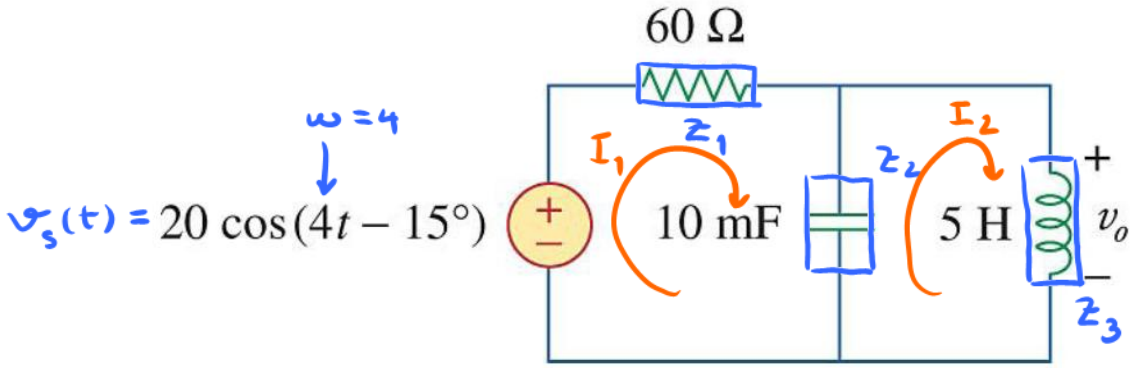
Course Title: *Basic Electrical Engineering*

Name	ID

Instructions

1. Separate into groups of no more than three persons.
2. Closed book. Closed notes.
3. Only one submission is needed for each group. Late submission will not be accepted.
4. **Do not panic.**

1. Determine $v_o(t)$ in the circuit below using mesh analysis. Also, find all mesh currents (in the counterclockwise direction).



$V_s = 20 \angle -15^\circ$

$= 19.3 - 5.18j$

$Z_1 = 60$

$Z_2 = \frac{1}{j\omega C} = \frac{1}{j \times 4 \times 10 \times 10^{-3}}$

$= \frac{100}{4j} = \frac{25}{j} = -25j$

$Z_3 = j\omega L = j \times 4 \times 5$

$= 20j$

Mesh Analysis

Loop 1:

$$V_s - I_1 Z_1 - (I_1 - I_2) Z_2 = 0$$

$$(Z_1 + Z_2) I_1 - Z_2 I_2 = V_s \quad \ldots (1)$$

Loop 2:

$$-(I_2 - I_1) Z_2 - I_2 Z_3 = 0$$

$$I_2 (Z_2 + Z_3) = I_1 Z_2$$

$$I_1 = I_2 \frac{Z_2 + Z_3}{Z_2} \quad \ldots (2)$$

$$\frac{Z_2 + Z_3}{Z_2} = \frac{-25j + 20j}{-25j} = \frac{-5j}{-25j} = \frac{1}{5}$$

Plugging the expression of I_1 from (2) into (1), we have

$$\left(\frac{(Z_1 + Z_2)(Z_2 + Z_3)}{Z_2} - Z_2 \right) I_2 = V_s$$

$$\frac{Z_1 Z_2 + Z_2^2 + Z_1 Z_3 + Z_2 Z_3 - Z_2^2}{Z_2} I_2 = V_s$$

Therefore,

$$I_2 = \frac{Z_2 V_s}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3} = 0.2358 - 0.8244j = \boxed{0.85 \angle -74^\circ}$$

From (2),

$$I_1 = \frac{Z_2 + Z_3}{Z_2} I_2 = \frac{1}{5} \times I_2$$

$$= \frac{(Z_2 + Z_3) V_s}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3} = 0.0472 - 0.165j = \boxed{0.17 \angle -74^\circ}$$

By Ohm's law

$$V_o = I_2 Z_3$$

$$= \frac{Z_2 Z_3 V_s}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3} = 16.48 + 4.72j = 17.15 \angle 15.96^\circ$$

Finally, we convert the phasor V_o back to $v_o(t)$:

$$\boxed{v_o(t) = 17.15 \cos(4t + 15.96^\circ)}$$