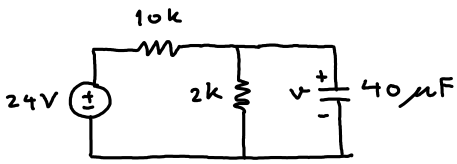
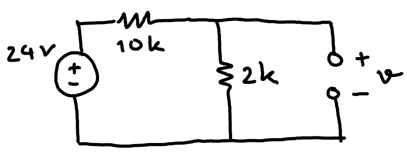
(7.6) The first step is to figure out the different circuits that are resulted from the move of the switch.

Case 1 : $t < 0$



The question asks for $v(t)$ for $t \geq 0$. We use this case-1 circuit to find $v(0^-)$ which is the same as $v(0)$ because there can't be any voltage jump across the capacitor.

Note, however, that the circuit has been put in this form for a long time. So, it is safe to assume that it has already reached the steady-state. In which case, the capacitor would act like an open circuit.



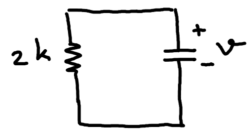
$v(0^-) = \frac{2}{10+2} \times 24 = 4$ V

voltage divider

Again, there can be no jump in the voltage across the capacitor. Hence,

$v(0) = v(0^-) = 4$ V.

Case 2 ; $t > 0$



For this circuit, we already know (from lecture) that

$v(t) = v(0)\, e^{-t/\tau}, \; t \geq 0.$

Case-1 gives $v(0) = 4$ V

$\tau = R \times C = 2k \times 40\,\mu F$

$= 80 \times 10^{-3}$ sec.

$\frac{1}{\tau} = \frac{100\cancel{0}}{8\cancel{0}} = 12.5$

$$\boxed{v(t) = 4e^{-12.5t}, \; t \geq 0}$$

Alternatively, we can find $v(\infty)$ and use a more general formula. At time t approaching ∞, the circuit should already be at its steady state. Therefore, the capacitor will act like an open circuit (again).

In this case, because there is no source driving the circuit,

$v(\infty) = 0$

We may plug this into the formula

$v(t) = v(\infty) + \left(v(0) - v(\infty)\right) e^{-t/\tau}, \; t \geq 0$

and get

$v(t) = 0 + (4 - 0)\, e^{-12.5t}, \; t \geq 0$

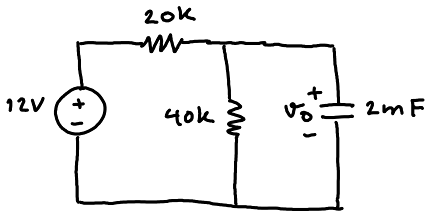
$= 4e^{-12.5t}, \; t \geq 0$

which is the same as what we found above.

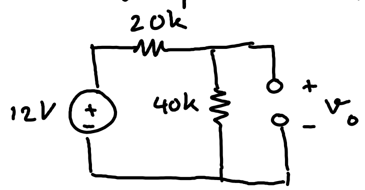
(7.7) Here. I will omit some of the explanation because the

(7.7) Here, I will omit some of the explanation because the approach is the same as the previous question.

Case 1: $t < 0$



At $t = 0^-$, the capacitor becomes open circuit.

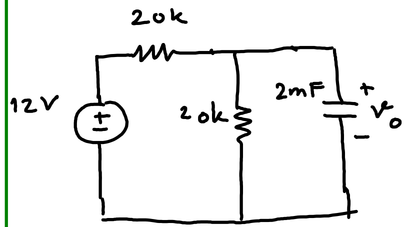


$$v_o(0^-) = \frac{4\cancel{0}}{\cancel{4\cancel{0}+2\cancel{0}}} \times \cancel{12}^{\,2}$$
$$= 8 \text{ V}$$

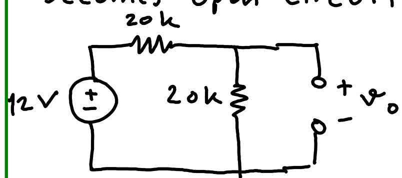
No jump in capacitor voltage:

$$v_o(0) = v_o(0^-) = 8\text{V}$$

Case 2: $t > 0$

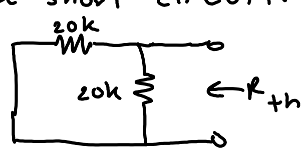


At $t = \infty$, the capacitor becomes open circuit



$$v_o(\infty) = \frac{12}{2} = 6 \text{ V.}$$

To find R_{th}, we replace the voltage source by a short circuit:



$$R_{th} = \frac{20k}{2} = 10\text{ k}\Omega$$

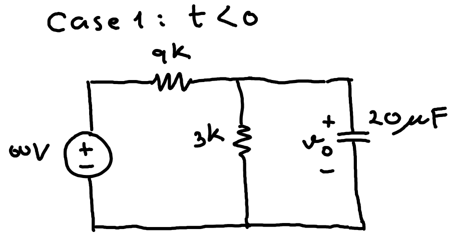
$$\tau = R_{th} \times C = 10k \times 2m$$
$$= 20 \text{ sec.}$$

$$v_o(t) = v_o(\infty) - (v_o(0) - v_o(\infty))\, e^{-t/\tau}, \quad t > 0$$
$$= 6 + (8 - 6)\, e^{-t/20} \text{ V}, \quad t > 0$$
$$= \boxed{6 + 2e^{-t/20} \text{ V}, \quad t > 0}$$

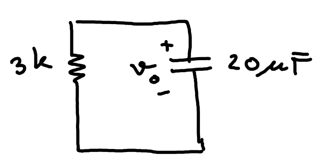
(7.10) Note that there are two parts for this question.

Part 1: The usual finding of voltage across the capacitor. Note that the setup is the same as Q7.6.

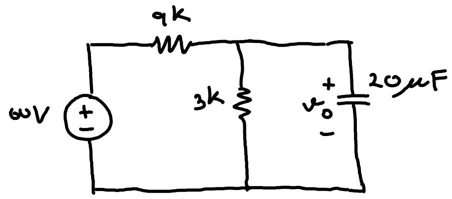
Case 1: $t < 0$



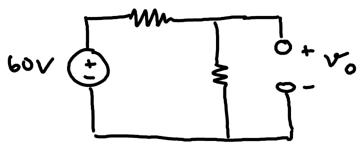
Case 2: $t > 0$



Case 1: $t<0$



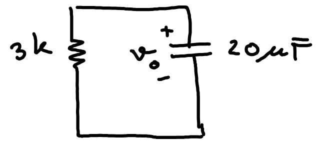
At $t=0^-$:



$$v_o(0^-) = \frac{3}{\cancel{3+9}} \times \cancel{60}^{5} = 15\text{ V}$$

$$v_o(0) = v_o(0^-) = 15\text{ V}$$

Case 2: $t>0$



$$v_o(t) = v_o(0)\, e^{-t/\tau}, \quad t>0$$

$$\tau = R \times C$$
$$= 3k \times 20\mu$$
$$= 60 \times 10^{-3} \text{ sec}$$

$$\frac{1}{\tau} = \frac{100\cancel{0}}{6\cancel{0}} = \frac{50}{3}$$

$$\boxed{v_o(t) = 15 \times e^{-\frac{50}{3}t} \text{ V}, \quad t>0}$$

Part 2: Find t such that $v_o(t) = \frac{1}{3} v_o(0)$

$$\Downarrow$$

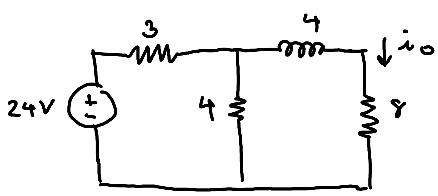
$$e^{-5\cancel{0}/3\, t} = \frac{1}{3}$$

$$\frac{50}{3} t = \ln 3$$

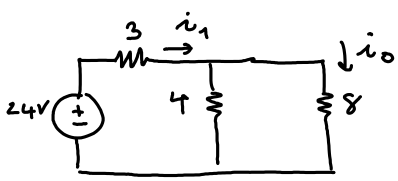
$$t = \boxed{\frac{3\ln 3}{50} \approx 65.9\text{ ms}}$$

7.11

Case 1: $t<0$



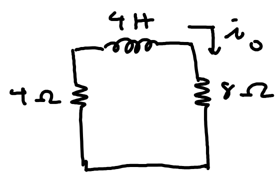
At $t=0^-$, inductor becomes short circuit.



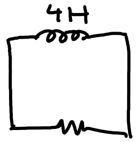
$$i_1 = \frac{24}{3+4//8} = \frac{72}{17}$$

$$i_o = \frac{4}{4+8} \times i_1 = \frac{4}{\cancel{12}} \times \frac{\cancel{72}^{6}}{17} = \frac{24}{17}$$

Case 2: $t>0$



$$\parallel\!\!|$$



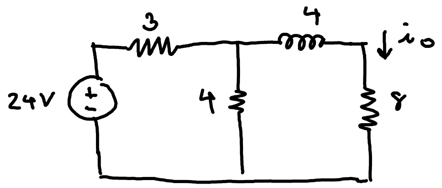
12 Ω

$$\tau = \frac{L}{R} = \frac{4}{12} = \frac{1}{3} \text{ sec.}$$

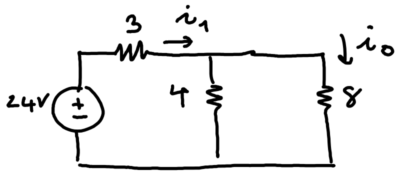
$$\boxed{i_o(t) = \frac{24}{17} \times e^{-3t}, \quad t>0}$$

≈ 1.412

Case 1 : $t < 0$



At $t = 0^-$, inductor becomes short circuit.

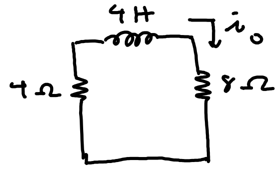


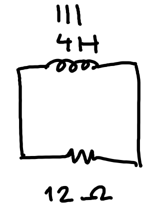
$$i_1 = \frac{24}{3 + 4/\!/8} = \frac{72}{17}$$

$$i_o = \frac{4}{4+8} \times i_1 = \frac{4}{12} \times \frac{72}{17} = \frac{24}{17}$$

So, $i_o(0) = i_o(0^-) = \frac{24}{17}$

case 2 : $t > 0$





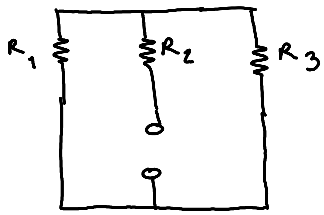
$$\tau = \frac{L}{R} = \frac{4}{12} = \frac{1}{3} \text{ sec.}$$

$$\boxed{i_o(t) = \frac{24}{17} \times e^{-3t}, \ t > 0}$$

≈ 1.412

7.16

(a)



$$R_{th} = R_2 + (R_1 /\!/ R_3)$$

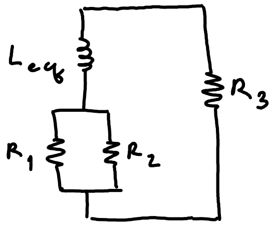
$$= R_2 + \frac{R_1 R_3}{R_1 + R_3}$$

$$= \frac{R_2 R_1 + R_2 R_3 + R_1 R_3}{R_1 + R_3}$$

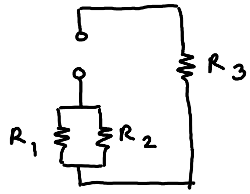
$$\tau = \frac{L}{R_{th}} = \boxed{\frac{L \times (R_1 + R_3)}{R_1 R_2 + R_2 R_3 + R_1 R_3}}$$

(b) First, the two inductor in parallel can be combined into

$$L_{eq} = L_1 /\!/ L_2 = \frac{L_1 L_2}{L_1 + L_2}$$



Next, we find R_{th}



$$R_{th} = R_1 /\!/ R_2 + R_3$$

$$= \frac{R_1 R_2}{R_1 + R_2} + R_3$$

$$= \frac{R_1 R_2 + R_3 (R_1 + R_2)}{R_1 + R_2}$$

$$\tau = \frac{L_{eq}}{R_{th}} = \boxed{\frac{L_1 L_2 (R_1 + R_2)}{(L_1 + L_2)(R_1 R_2 + R_1 R_3 + R_2 R_3)}}$$

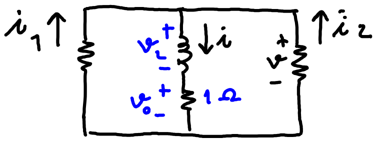
(7.22) Using what we got in 7.16a,

we have $R_{th} = 1 + 5 // 20\ \Omega$
$= 1 + 4 = 5\ \Omega$

$$\tau = \frac{L}{R_{th}} = \frac{2}{5} \text{ sec.}$$

$$i(t) = i_L(t) = i_L(0)\, e^{-t/\tau} = \boxed{20 \times e^{-2.5t}\ \text{A},\ t > 0}$$

Let $i_2(t)$ be the current that passes through the 20Ω resistor.



$$i_2(t) = \underbrace{\frac{5}{5+20}}_{\text{current divider}} \times i(t) = \frac{5}{25} i(t) = \frac{1}{5} i(t)$$

$$= 4e^{-2.5t}\ \text{A},\quad t > 0$$

$$v(t) = -20\,\Omega \times i_2(t) = \boxed{-80\, e^{-2.5t}\ \text{V},\ t > 0}$$

Alternatively, we find $v(t)$ via $v_L(t) + v_o(t)$.

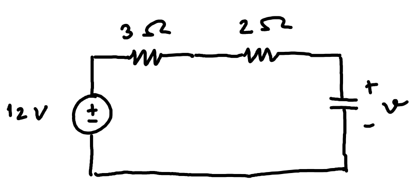
$$L\frac{di}{dt} = 2 \times 20 \times (-2.5)\, e^{-2.5t} = -100\, e^{-2.5t}$$

$$i \times 1\Omega = 20 \times e^{-2.5t} \times 1 = 20 \times e^{-2.5t}$$

The sum is
$-80\, e^{-2.5t}$
which is the same as the answer above.

(7.40)

(a) Case 1: $t < 0$



As $t \to 0^-$, capacitor

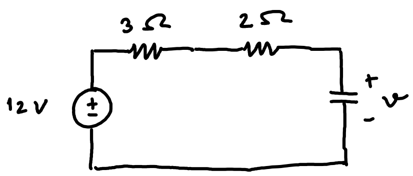
Case 2: $t > 0$

Initial condition:

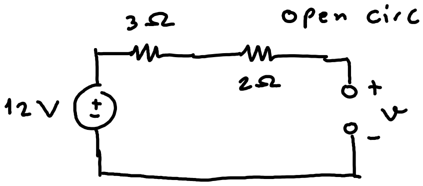
$$v(0) = v(0^-) = 12\ \text{V}$$

no voltage jump across capacitor.

(a) Case 1: $t < 0$



As $t \to 0^-$, capacitor $\downarrow$ open circuit



$v(0^-) = 12\,V$

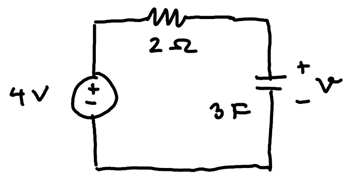
(There is no current through the 3Ω and 2Ω and hence there is no voltage drop.)

Case 2: $t > 0$

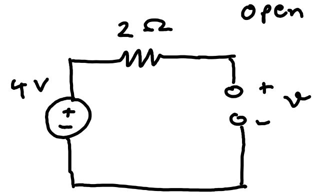
Initial condition:

$v(0) = v(0^-) = 12\,V$

no voltage jump across capacitor.



As $t \to \infty$, capacitor $\downarrow$ open circuit



$R_{th} = 2\,\Omega$

$V_{th} = 4\,V$

$v(\infty) = 4V$

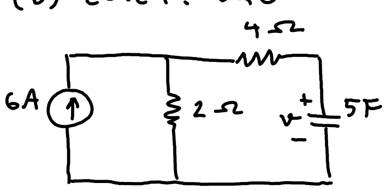
no current through 2Ω.

$\tau = R_{th} \times C = 2 \times 3 = 6$ sec.

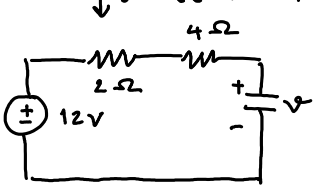
$v(t) = 4 + (12-4)e^{-t/6}\ V$

$$\boxed{v(t) = \begin{cases} 12\ V, & t < 0 \\ 4 + 8e^{-t/6}\ V, & t \geq 0 \end{cases}}$$

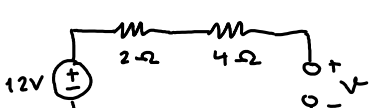
(b) case 1: $t < 0$



$\downarrow$ source transformation



As $t \to 0^-$, capacitor $\downarrow$ open circuit

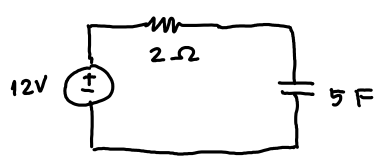


case 2: $t > 0$

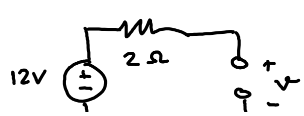
Initial condition: $v(0) = v(0^-) = 12\,V$

no voltage jump across capacitor

When $t > 0$, there is a short connection across the 4Ω. Hence, we can ignore the 4Ω.



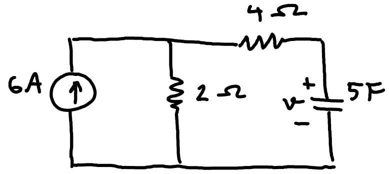
As $t \to \infty$, capacitor $\to$ open circuit.



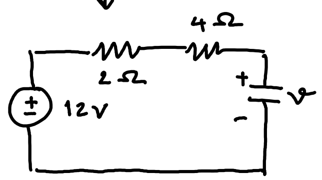
$R_{th} = 2\,\Omega$

$V_{th} = 12\,V$

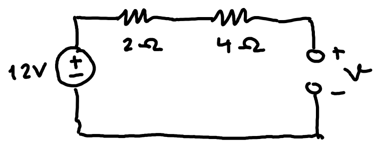
(b) case 1: $t<0$



↓ source transformation



As $t \to 0^-$, capacitor → open circuit



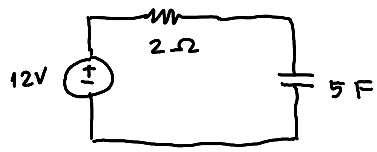
$v(0^-) = 12V$

case 2: $t>0$

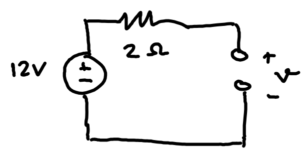
Initial condition: $v(0) = v(0^-) = 12V$

no voltage jump across capacitor

when $t>0$, there is a short connection across the 4Ω. Hence, we can ignore the 4Ω.



As $t \to \infty$, capacitor → open circuit.



$R_{th} = 2\Omega$

$V_{th} = 12V$

$v(\infty) = 12V$

$\tau = R_{th} \times C = 2 \times 5 = 10$ s.

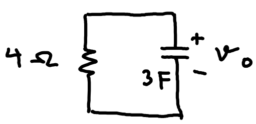
$v(t) = 12 + \cancel{(12-12)} e^{-t/10} = 12V$

$\to 0$

$\boxed{v(t) = 12\ V}$ for all t

7.42

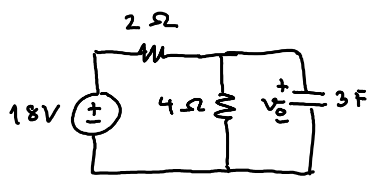
case 1: switch open



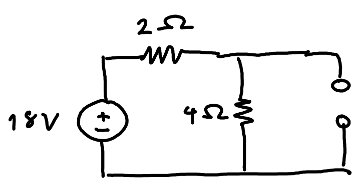
$\tau = 3 \times 4 = 12$ s

$v_o(\infty) = 0$

case 2: switch closed



As $t \to \infty$, capacitor → open circuit



$R_{th} = 2 // 4$

$= \frac{2 \times 4}{6} = \frac{4}{3}\ \Omega$

$V_{th} = \frac{4}{2+4} \times 18 = 12V$

$\tau = R_{th} \times C = \frac{4}{3} \times 3 = 4$ sec.

$v_o(\infty) = 12$ V.

(a) Switch open → Switch closed

$v_o(0^-) = 0$

$v_o(0) = v(0^-) = 0\ V$

$v_o(t) = v_o(\infty) + (v_o(0) - v_o(\infty))e^{-t/\tau}, \quad t > 0$

$= 12 + (0 - 12)e^{-t/4}$

$= 12(1 - e^{-t/4})$

$$\boxed{v_o(t) = \begin{cases} 0, & t < 0 \\ 12(1 - e^{-t/4}), & t \geq 0 \end{cases}}\ V$$

(b) Switch open $\leftarrow$ Switch closed.

$v_o(0) = v_o(0^-) = 12\ V$ $\qquad$ $v_o(0^-) = 12\ V$

For $t > 0$,

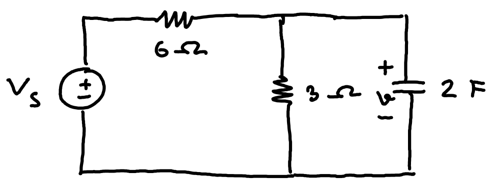
$v_o(t) = v_o(\infty) + (v_o(0) - v_o(\infty))e^{-t/\tau}$

$= 0 + (12 - 0)e^{-t/12}$

$= 12e^{-t/12}\ V$

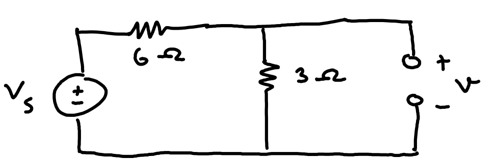
$$\boxed{v_o(t) = \begin{cases} 12\ V, & t < 0 \\ 12e^{-t/12}\ V, & t \geq 0 \end{cases}}$$

7.44 The circuits for $t<0$ and $t>0$ have the same form. So, to save space, we will analyze them together.



$V_s = \begin{cases} 30\ V, & t < 0 \\ 12\ V, & t > 0 \end{cases}$

When we leave the circuit in this form for a long time, capacitor $\rightarrow$ open circuit



$R_{th} = 6 // 3 = \frac{\cancel{6}^{2} \times \cancel{3}}{\cancel{9}_{\cancel{3}}} = 2\ \Omega$

$V_{th} = \frac{3}{6+3} \times V_s = \frac{1}{3} V_s$

$\tau = R_{th} \times C = 2 \times 2 = 4$ sec.

Case 1 : $t < 0$

$V_s = 30\ V$

$v(0^-) = V_{th} = \frac{1}{3} \times 30 = 10\ V$

Case 2 : $t > 0$, $V_s = 12\ V$

Initial condition:

$v(0) = v(0^-) = 10\ V$

↑

no jump in voltage
value across the capacitor.

$v(\infty) = V_{th} = \frac{1}{3} \times 12 = 4\ V$

$v(t) = 4 + (10 - 4)\, e^{-t/4}\ V$

$= 4 + 6\, e^{-t/4}\ V$

$i(t) = C \frac{dv}{dt}$

$= 2 \left(6 \times -\frac{1}{4}\, e^{-t/4} \right)$

$= \boxed{-3\, e^{-t/4}\ A}$