HW7

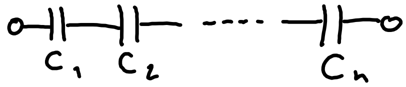
Friday, January 29, 2010

10:54 AM

The first four exercises in this HW deal with capacitor combination. To simplify our notation, we will use

$$\text{series}(C_1, C_2, \ldots, C_n)$$

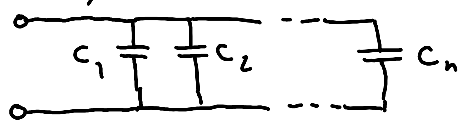
to mean the combined value of $C_1, C_2, \ldots, C_n$ when they are all connected in series:



We will use

$$\text{parallel}(C_1, C_2, \ldots, C_n)$$

to mean the combined value of $C_1, C_2, \ldots, C_n$ when they are all connected in parallel:



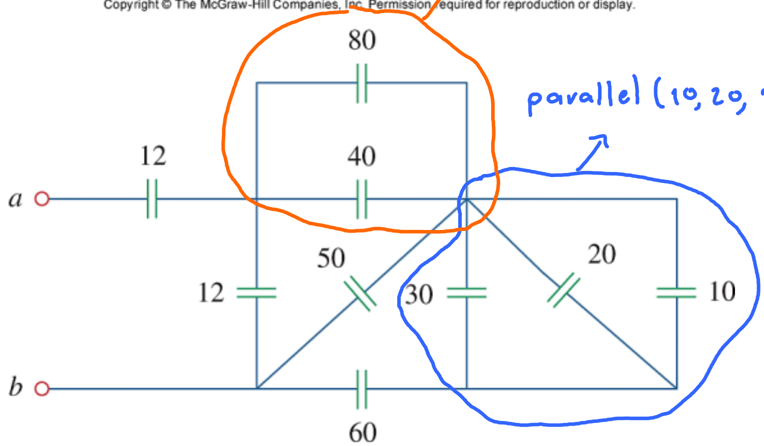
We know that $\text{series}(C_1, \ldots, C_n) = \left(\frac{1}{C_1} + \cdots + \frac{1}{C_n}\right)^{-1}$

$$\text{parallel}(C_1, \ldots, C_n) = C_1 + \cdots + C_n$$

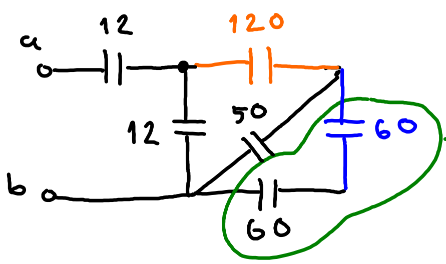
6.19

parallel(80, 40) = 120

parallel(10, 20, 30) = 60



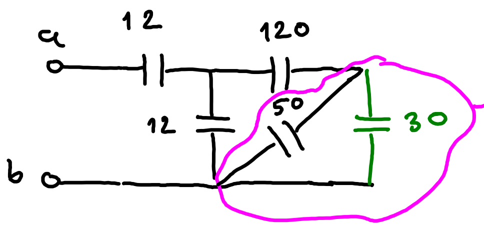
$\equiv$



$\rightarrow \text{series}(60, 60) = \dfrac{1}{\frac{1}{60} + \frac{1}{60}}$

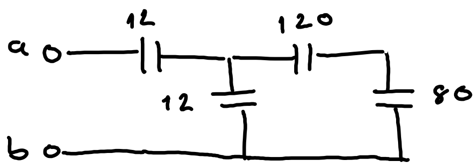
$= \dfrac{60}{2} = 30$

$\equiv$



parallel $(30, 50) = 80$

$\equiv$



At this point, I'm too lazy to draw more pictures. Our simplified combination above is easy enough to be represented in a one-line expression:

$$C_{ab} = \text{series}\left(12, \text{parallel}\left(12, \underbrace{\text{series}(120, 80)}_{\frac{120\times 80}{120+80} = \frac{\cancel{120}^{6}\times \cancel{80}}{\cancel{200}} = 48}\right)\right)$$

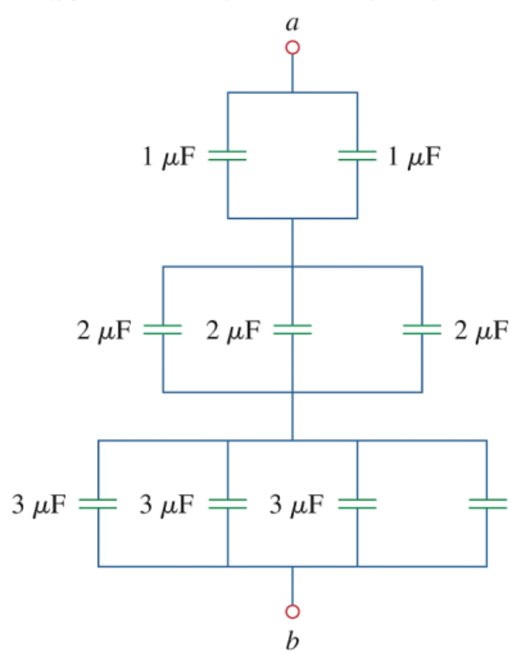
$$\underbrace{48 + 12 = 60}$$

$$\frac{60\times 12}{60+12} = \frac{\cancel{60}^{10}\times \cancel{12}}{\cancel{72}_{6}} = 10$$

$= \boxed{10\ \mu F}$

don't forget the unit!!

6.20

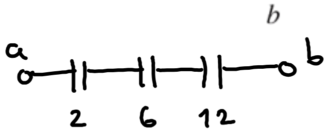


parallel $(1, 1) = 2$

parallel $(2, 2, 2) = 6$

parallel $(3, 3, 3, 3) = 3\times 4$
$= 12$

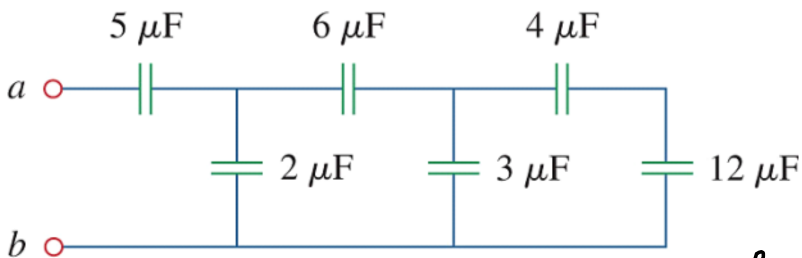
$\equiv$



$$C_{ab} = \frac{1}{\frac{1}{2}+\frac{1}{6}+\frac{1}{12}} = \frac{2}{1+\underbrace{\frac{1}{3}+\frac{1}{6}}_{\frac{1}{2}}} = \boxed{\frac{4}{3}\ \mu F}$$

6.21

6.21



$$\frac{4\times12}{4+12} = \frac{\cancel{4}\times\cancel{12}^{3}}{\cancel{16}_{4}} = 3\,\mu F$$

$$3+3 = 6\,\mu F$$

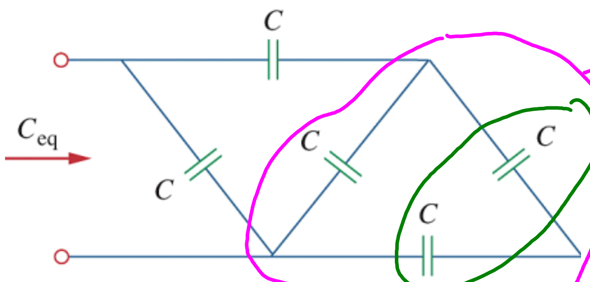
$$\frac{6\times6}{6+6} = \frac{1}{2}\times6 = 3\,\mu F$$

$$2+3 = 5\,\mu F$$

$$\frac{5\times5}{5+5} = \frac{1}{2}\times5 = \boxed{2.5\,\mu F}$$

6.29

$$\frac{1\times\frac{3}{2}}{1+\frac{3}{2}} = \frac{3}{2+3} = \frac{3}{5}$$



(a)

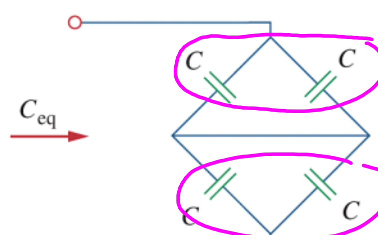
Parallel$(C, \frac{C}{2}) = \frac{3C}{2}$

series$(C, C) = \frac{C}{2}$

C_{eq} = parallel$(C$, series$(C, \frac{C}{2}))$

= parallel$(C, \frac{3}{5}C)$

$= C + \frac{3}{5}C = \boxed{\frac{8}{5}C}$



(b)

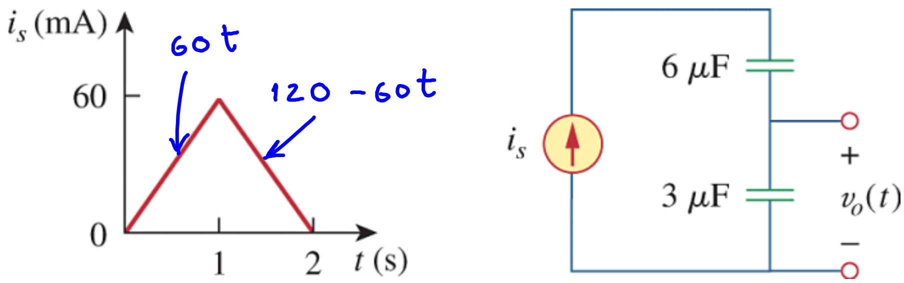
parallel$(C, C) = C + C = 2C$

parallel$(C, C) = C + C = 2C$

C_{eq} = series$(2C, 2C) = \underline{2C} = C$

$$C_{eq} = \text{series}(2C, 2C) = \frac{2C}{2} = C$$

6.30



For capacitor, $\underbrace{i_c(t)}_{= i_s(t)} = C \dfrac{d\,v_c(t)}{dt}$

$v_c(t) = v_o(t)$

$$v_o(t) = v_c(t) = \frac{1}{C}\int_0^t i_c(x)\,dx + v_o(0)$$

$$= \frac{1}{C}\int_0^t i_s(x)\,dx + 0$$

when there is no charge $(q(0) = 0)$ in the capacitor,

$$\underbrace{v_c(0)}_{= v_o(0)} = \frac{q(0)}{C} = \frac{0}{C} = 0$$

Note that $i_s(x) = \begin{cases} 60x \text{ mA}, & 0 \le x \le 1 \\ 120 - 60x \text{ mA}, & 1 \le x \le 2 \end{cases}$

So, for $0 \le t \le 1$

$$v_o(t) = \frac{1}{C}\int_0^t 60x\,dx \times 10^{-3} = \frac{1}{C}\,\cancel{60}^{\,30}\,\frac{t^2}{\cancel{2}} \times 10^{-3}$$

At $t = 1$,

$$v_o(1) = \frac{1}{C}\,30 \times 10^{-3}.$$

For $1 \le t \le 2$

$$v_o(t) = v_o(1) + \frac{1}{C}\underbrace{\int_1^t 120 - 60x\,dx}\times 10^{-3}$$

$$120(t-1)-30(t^2-1)$$

$$= 120t - 120 + 30 - 30t^2$$

$$= 30(4t - 3 - t^2)$$

Now, we combine all the cases:

$$v_o(t) = \frac{1}{C} \times 10^{-3} \times \begin{cases} 30t^2, & 0 \leq t \leq 1 \\ 30 + 30(4t-3-t^2), & 1 \leq t \leq 2 \end{cases}$$

$$= \underbrace{\frac{1}{C} \times 30 \times 10^{-3}}_{\frac{1}{\cancel{3} \times 10^{-6}} \times \cancel{30}^{10} \times 10^{-3} = 10 \times 10^{3}} \times \begin{cases} t^2, & 0 \leq t \leq 1 \\ 4t-2-t^2, & 1 \leq t \leq 2 \end{cases}$$

$$= \boxed{\begin{cases} 10t^2, & 0 \leq t \leq 1 \\ 40t - 20 - 10t^2, & 1 \leq t \leq 2 \end{cases} \quad kV}$$