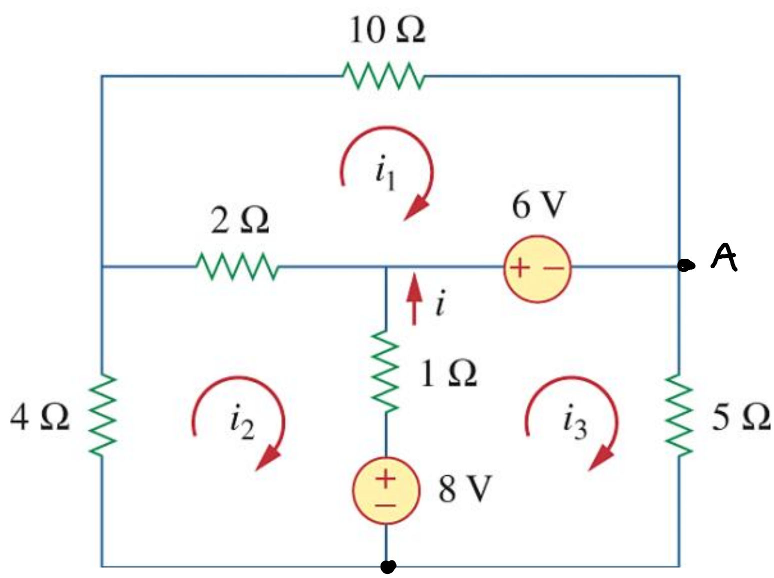
HW3, Chapter 3

Wednesday, December 02, 2009

2:38 PM

3.41



For loop 1: We apply KVL starting from node A:

$$+6 - 2\times(\underbrace{i_1 - i_2}) - 10\, i_1 = 0$$

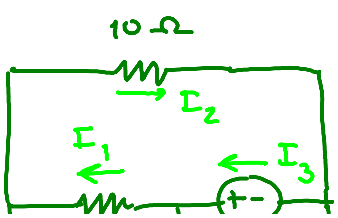
Moving from − to + terminal of a 6V voltage source means I gain 6 V.

Moving pass the resistor means I lose IR volts

The current here must be $i_1 - i_2$ because both i_1 and i_2 passes this 2Ω resister.

If you feel uncomfortable with the above method for getting the equation, we may get the same equation via another method:

In loop 1, we have



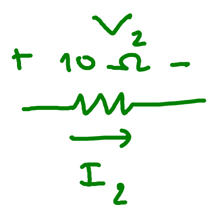
We first need to find the currents passing through these elements : I_1, I_2, I_3

It turns out that I will apply KVL later and hence I don't really care about the I_3. So, we will focus only on I_1 and I_2.

Notice how the mesh currents are defined. We see that I_2 is the same as i_1. However, for I_1 we hae two mesh currents i_1 and i_2 going in opposite direction. For my I_1 above, I have

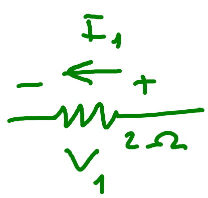
$$I_1 = i_1 - i_2 .$$

Next, by Ohm's law, we have



$$V_2 = I_2 \times 10$$
$$= i_1 \times 10 \ V$$

and

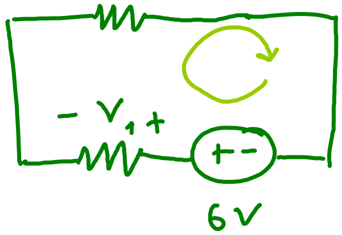


$$V_1 = I_1 \times 2$$
$$= (i_1 - i_2) \times 2 \ V$$

Note that I don't have any minus sign in my Ohm's low because I defined the directions of my currents and polarities of my voltages so that the current flows from a higher potential (+) to a lower potential (−).

Return to loop 1, we now have

$+ V_2 -$



So, by KVL, we have

$$-6 + V_1 + V_2 = 0.$$

(here I use the trick that we discussed in class, the sign on each voltage is the polarity of the terminal encountered first as we travel around the loop)

Plugging the values for V_1 and V_2, we then have

$$-6 + (i_1 - i_2) \times 2 + i_1 \times 10 = 0$$

which is the same equation that we got earlier.

Once you did this KVL equation reading for many times, you should be able to "read" the equation directly from a loop without thinking much.

with correct signs

For loop 2, starting from the bottom, we have

$$-4 \times i_2 - 2 \times (i_2 - i_1) - 1 \times (i_2 - i_3) - 8 = 0$$

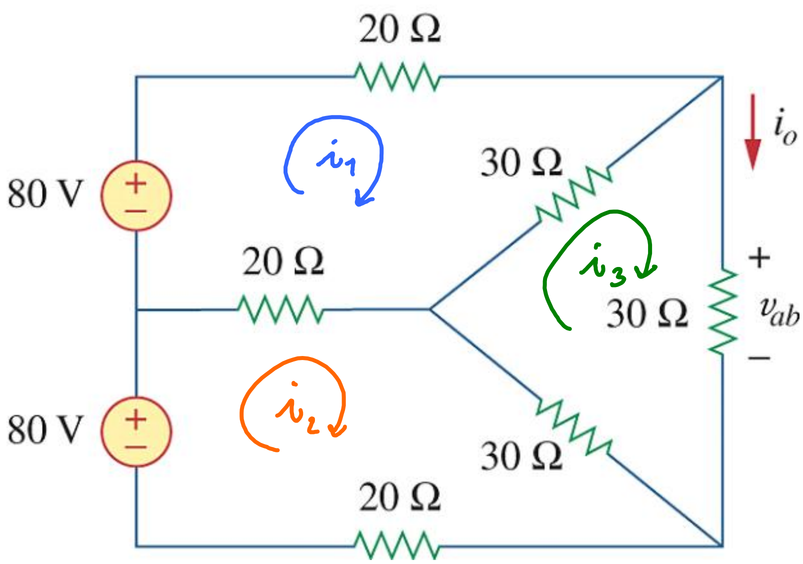
For loop 3, starting from A, we have

$$-5 i_3 + 8 - (i_3 - i_2) \times 1 - 6 = 0$$

$$\Rightarrow i_1 = \frac{77}{234}, \quad i_2 = -\frac{40}{39}, \quad i_3 = \frac{19}{117}$$

Therefore, $i = i_3 - i_2 = \boxed{\frac{139}{117} \approx 1.19 \text{ A}}$

3.43



Loop 1 : $80 - i_1 \times 20 - 30 \times (i_1 - i_3) - 20(i_1 - i_2) = 0$

Loop 2 : $80 - (i_2 - i_1) \times 20 - (i_2 - i_3) \times 30 - 20 i_2 = 0$

Loop 3 : $-30(i_3 - i_2) - 30(i_3 - i_1) - 30 i_3 = 0$

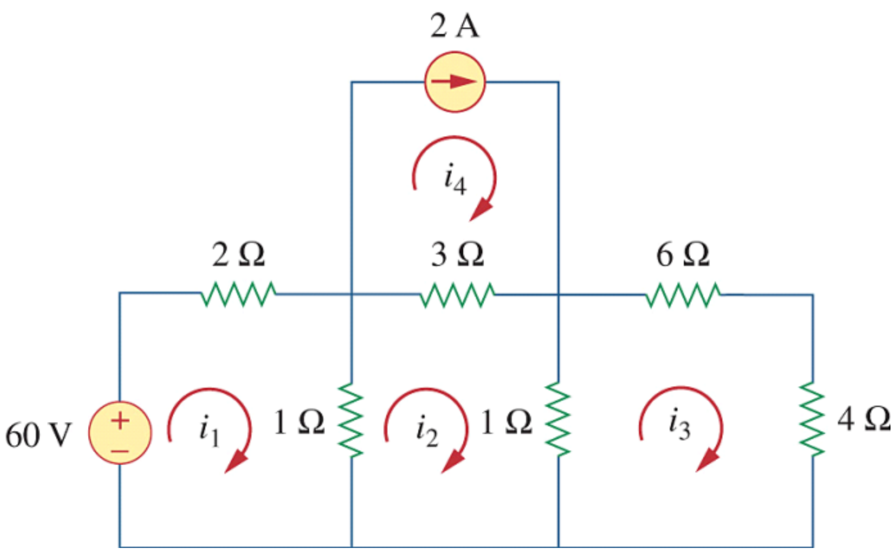
$\Rightarrow i_3 = \frac{16}{9} \quad \left(i_1 = i_2 = \frac{8}{3} \right)$

Note that $i_o = i_3$.

So, $i_o = \boxed{\frac{16}{9} A \approx 1.78 A}$

$v_{ab} = i_o \times 30 = \frac{16}{\cancel{9}_{3}} \times \cancel{30} = \boxed{\frac{160}{3} V \approx 53.33 V}$

3.46



In this problem, because i_4 is the only current that passes through 2 A current source. We automatically get $\boxed{i_4 = 2A.}$

Loop 1 : $+60 - 2i_1 - (i_1 - i_2) = 0$

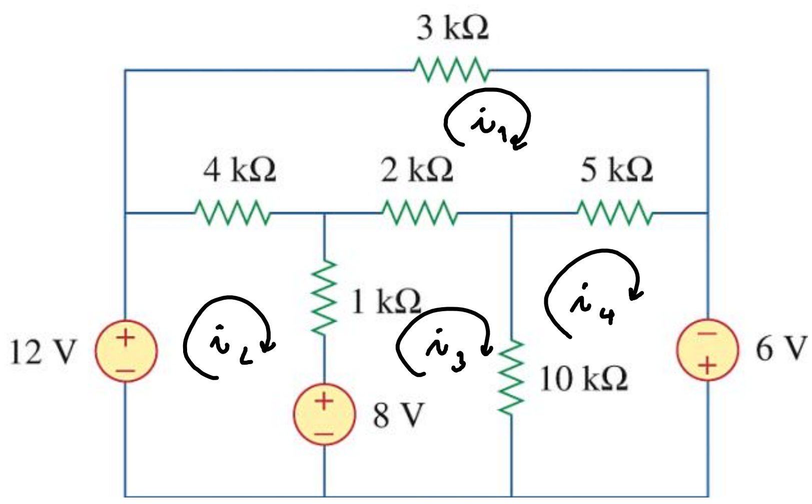
Loop 2 : $-(i_2 - i_1) - 3(i_2 - \underset{\uparrow 2}{i_4}) - (i_2 - i_3) = 0$

Loop 3 : $-(i_3 - i_2) - 6i_3 - 4i_3 = 0$

$$\Rightarrow i_1 = \frac{3306}{151}, \quad i_2 = \frac{858}{151}, \quad i_3 = \frac{78}{151}$$

$$\boxed{i_1 \approx 21.89A, \; i_2 \approx 5.68A, \; i_3 \approx 0.52\,A}$$

3.48



Loop1 : $-3i_1 - 4(i_1 - i_2) - 2(i_1 - i_3) - 5(i_1 - i_4) = 0$

Loop2 : $12 - 4(i_2 - i_1) - (i_2 - i_3) - 8 = 0$

Loop3 : $8 - (i_3 - i_2) - 2(i_3 - i_1) - 10(i_3 - i_4) = 0$

Loop4 : $-10(i_4 - i_3) - 5(i_4 - i_1) + 6 = 0$

$$\Rightarrow \quad i_1 = 6\,mA, \quad i_2 = \frac{166}{23}\,mA, \quad i_3 = \frac{186}{23}\,mA, \quad i_4 = \frac{896}{115}\,mA$$

currents are in mA because resistances are in kΩ.

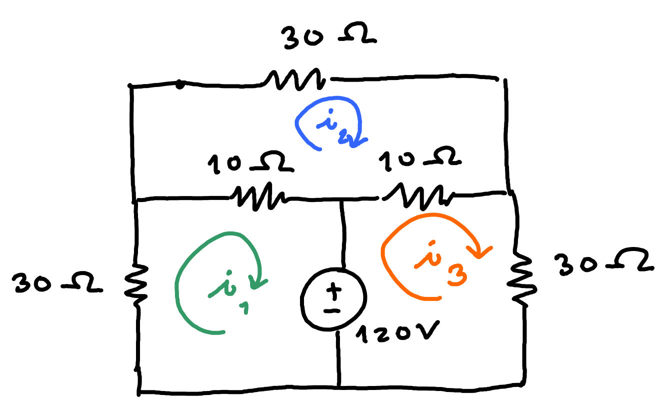
The current through the 10 kΩ is

$$i_4 - i_3 = \frac{896}{115} - \frac{186}{23} = -0.296\,mA$$

pointing up

So, it is $\boxed{0.296\ mA}$ pointing down.

3.58



By KVL

Loop 1 : $-3\cancel{0}i_1 - (i_1 - i_2)\times 1\cancel{0} - 12\cancel{0} = 0$

$$-4i_1 + i_2 = 12 \qquad (1)$$

Loop 2 : $-1\cancel{0}(i_2 - i_1) - i_2(3\cancel{0}) - 1\cancel{0}\times(i_2 - i_3) = 0$

$$i_1 - 5i_2 + i_3 = 0 \qquad (2)$$

Loop 3 : $12\cancel{0} - 1\cancel{0}\times(i_3 - i_2) - 3\cancel{0}\times i_3 = 0$

$$i_2 - 4i_3 = -12 \qquad (3)$$

Solving (1), (2), (3), we get

$$\boxed{\begin{aligned} i_1 &= -3\,A \\ i_2 &= 0\,A \\ i_3 &= 3A \end{aligned}}$$