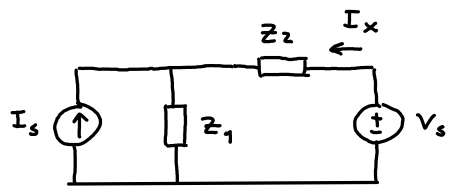
HW12

Sunday, February 28, 2010

10:57 PM

10.43



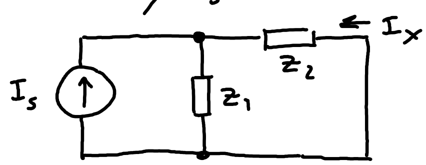
For this question, both i_s and v_s have the same frequency which is $\omega = 2$.

Hence, $Z_1 = j\omega L = j \times 2 \times 4 = 8j$

$$Z_2 = \frac{1}{j\omega C} + 3 = \frac{1}{j \times \cancel{2} \times \frac{1}{\cancel{8}_4}} + 3 = \frac{4}{j} + 3 = 3 - 4j$$

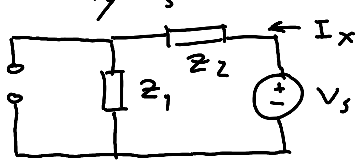
We also have $I_s = 5 \angle 10^\circ$ and $V_s = 10 \angle -60^\circ$.

Case 1: Only I_s is activated



$$I_x = -\frac{Z_1}{Z_1 + Z_2} I_s$$

Case 2: Only V_s is activated



$$I_x = \frac{V_s}{Z_1 + Z_2}$$

Now, because the two cases have the same frequency, we can combine the phasors.

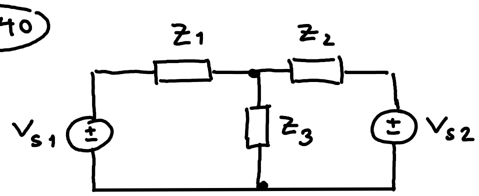
We can also convert them into sinusoids and then combine. The answer will be the same.

Caution If the frequencies are not the same, do not combine the phasors. You must convert them into sinusoids first.

$$I_x = -\frac{Z_1}{Z_1+Z_2} I_s + \frac{V_s}{Z_1+Z_2} = \frac{V_s - Z_1 I_s}{Z_1 + Z_2} = -6.26 - 7.68j = 9.9 \angle -129.17^\circ$$

$$i_x(t) = \boxed{9.9 \cos(2t - 129.17^\circ) \text{ A}}$$

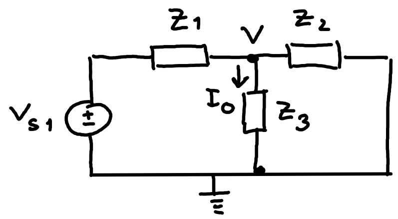
10.40



In this question, v_{s1} and v_{s2} do not have the same frequency. We need to be more careful with Z_3. This is because Z_3 is given in term of the inductance value. The impedance $Z_3 = j\omega L$

We need to be more careful with Z_3. This is because Z_b is given in term of the inductance value. The impedance $Z_3 = j\omega L$ will depend on the (angular) frequency ω.

Case 1 : Only V_{s1} is activated



$$V = \frac{Z_2 // Z_3}{Z_1 + Z_2 // Z_3} V_{s1}$$

$$= \frac{Z_2 Z_3}{Z_1(Z_2+Z_3) + Z_2 Z_3} \times V_{s1}$$

$$I_o = \frac{V}{Z_3} = \frac{Z_2}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3} \times V_{s1}$$

$\omega = 4$

$V_{s1} = 20$

$Z_1 = 4$

$Z_2 = 2$

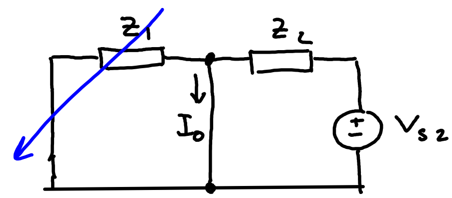
$Z_3 = j\omega L = j \times 4 \times 1 = 4j$

$I_o = 0.5 - 1.5j = 1.58 \angle -71.6^\circ \quad A$

$$i_o(t) = \boxed{1.58 \cos(4t - 71.6^\circ) + 8 \quad A}$$

Case 2 : Only V_{s2} is activated

Because V_{s2} is dc, we have $Z_3 = j \times 0 \times L = 0$

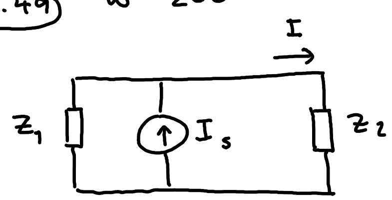


killed because of the short connection

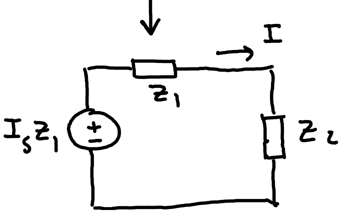
$$I_o = \frac{V_{s2}}{Z_2}$$

$$I_o = \frac{16}{2} = 8 \ A$$

10.49 $\quad \omega = 200$



Source transformation



$Z_1 = 5$

$$Z_2 = 3 + \frac{1}{j\omega C} + j\omega L$$

$$= 3 + \frac{1}{j\, 200 \times 1 \times 10^{-3}} + j \times 200 \times 5 \times 10^{-3}$$

$$= 3 + \frac{5}{j} + j = 3 - 5j + j = 3 - 4j$$

$I_s = 16 \angle 30^\circ - 90^\circ$

$= 16 \angle -60^\circ$

$= 8 - 13.86j$

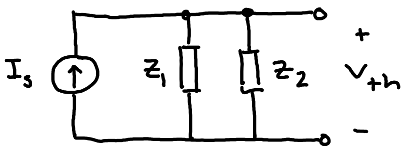
$$I = \frac{I_s Z_1}{Z_1 + Z_2} = 7.46 - 4.9j$$

$= 8.944 \angle -33.4^\circ$

$$I = \frac{I_s Z_1}{Z_1 + Z_2} = 7.46 - 4.9j$$
$$= 8.944 \angle -33.4^\circ$$

$$i(t) = \boxed{8.944 \cos(200t - 33.4^\circ)\ A}$$

10.58

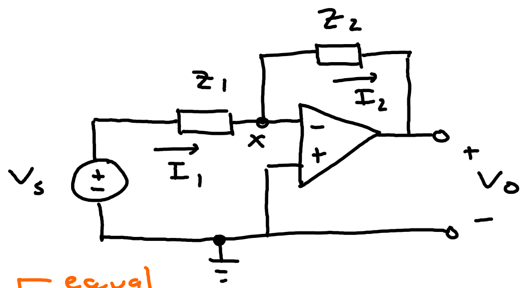


$$I_s = 2\angle 30^\circ = 1.732 + j$$
$$Z_1 = 8 - 6j$$
$$Z_2 = 10j$$

$$V_{th} = I_s \times (Z_1 // Z_2) = 12.32 + 18.7j = \boxed{22.36 \angle 56.6^\circ}$$

$$R_{th} = Z_1 // Z_2 = \frac{Z_1 Z_2}{Z_1 + Z_2} = 10 + 5j = \boxed{5(2+j)}$$

10.69



equal

$V_x = 0$ voltages at ideal op amp input terminals

$$\left.\begin{aligned} I_1 &= \frac{V_s}{Z_1} \\ I_2 &= \frac{V_x - V_o}{Z_2} = -\frac{V_o}{Z_2} \end{aligned}\right\} \qquad I_1 = I_2$$

no current for ideal opamp's input terminals

$$\frac{V_s}{Z_1} = -\frac{V_o}{Z_2}$$

$$\frac{V_o}{V_s} = -\frac{Z_2}{Z_1} = -\frac{R}{\frac{1}{j\omega C}} = \boxed{-j\omega RC}$$

this should not be bold format

$$v_s(t) = V_m \sin \omega t$$

$$V_s = V_m \angle -90^\circ$$

$$V_o = V_s \times -j\underbrace{\omega RC}_{=1 \text{ because } \omega = \frac{1}{RC}} = -jV_s = (1\angle -90^\circ)(V_m \angle -90^\circ)$$

$$= V_m \angle -180^\circ$$

$$v_o(t) = \boxed{V_m \cos(\omega t - 180^\circ)} = -V_m \cos \omega t$$

where $\omega = \frac{1}{RC}$.

11.1 $v(t) = 160 \cos(50t)$

$$i(t) = -20 \sin(50t - 30^\circ) = 20 \cos(50t - 30^\circ - 90^\circ + 180^\circ)$$
$$= 20 \cos(50t + 60^\circ)$$

Instantaneous power

$= 20 \cos(50t + 60^\circ)$

Instantaneous power

$$p(t) = v(t) \times i(t)$$
$$= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$
$$= 1600 \cos(60^\circ) + 1600 \cos(100t + 60^\circ)$$
$$= \boxed{800 + 1600 \cos(100t + 60^\circ)} \ W$$

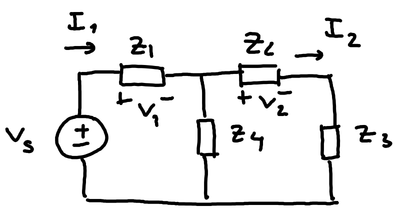
Average power

$$P = \boxed{800}$$

Alternatively, $P = \frac{1}{2} Re\{V I^*\} = \frac{1}{2} Re\{160 \times (20 \angle -60^\circ)\}$

$$= \frac{1}{2} \times 160 \times 20 \times \cos(-60^\circ) = 800.$$

11.5



$V_s = 16 \angle -40^\circ = 12.3 - 10.3j$

$\omega = 2$

$Z_1 = 1$

$Z_2 = 2$

$Z_3 = \frac{1}{j 2 \times \frac{1}{4}} = \frac{1}{\frac{1}{2}} \frac{2}{j} = -2j$

$Z_4 = j\omega L = j \times 2 \times 3 = 6j$

$$I_1 = \frac{V_s}{Z_1 + \underbrace{Z_4 // (Z_2 + Z_3)}_{3.6 - 1.2j}} = 3.04 - 1.44j = 3.366 \angle -25.38^\circ$$

$$P_1 = \frac{1}{2} Re\{I_1^* V_1\} = \frac{1}{2} Re\{I_1^* I_1 Z_1\} = \frac{1}{2} |I_1|^2 = 5.664 \ W$$

current divider

$$I_2 = \frac{Z_4}{(Z_2 + Z_3) + Z_4} \times I_1 = 4.514 + 0.093j = 4.515 \angle 1.2^\circ$$

$$P_2 = \frac{1}{2} Re\{I_2^* V_2\} = \frac{1}{2} Re\{I_2^* I_2 Z_2\} = \cancel{\frac{1}{2}} |I_2|^2 \times \cancel{2} = 20.39 \ W$$

The average power for capacitor or inductor is 0.

Therefore,

$$\boxed{\begin{aligned} P_{1\Omega} &= 5.664 \ W \\ P_{2\Omega} &= 20.39 \ W \\ P_{3H} &= 0 \ W \\ P_{0.25F} &= 0 \ W \end{aligned}}$$

$$\boxed{\begin{aligned} P_{1\Omega} &= 5.664 \text{ W} \\ P_{2\Omega} &= 20.39 \text{ W} \\ P_{3H} &= 0 \text{ W} \\ P_{0.25F} &= 0 \text{ W} \end{aligned}}$$