

Chapter 10, Solution 7.

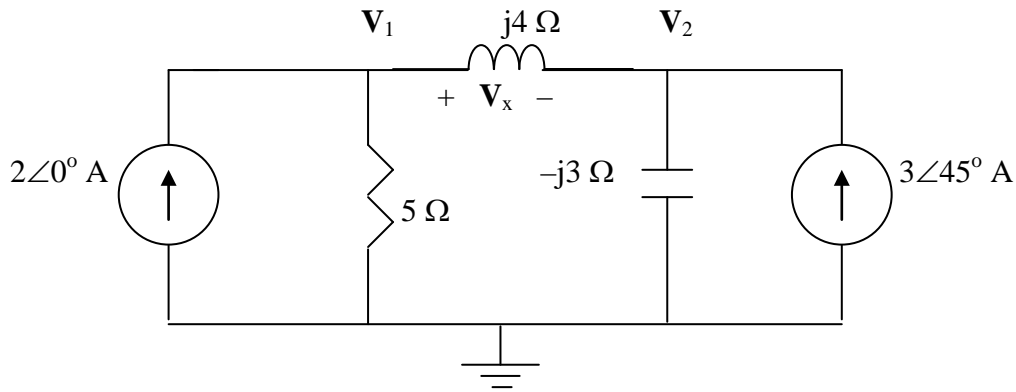
At the main node,

$$\frac{120\angle -15^\circ - V}{40 + j20} = 6\angle 30^\circ + \frac{V}{-j30} + \frac{V}{50} \longrightarrow \frac{115.91 - j31.058}{40 + j20} - 5.196 - j3 =$$
$$V\left(\frac{1}{40 + j20} + \frac{j}{30} + \frac{1}{50}\right)$$

$$V = \frac{-3.1885 - j4.7805}{0.04 + j0.0233} = \underline{124.08\angle -154^\circ \text{ V}}$$

Chapter 10, Solution 16.

Consider the circuit as shown in the figure below.



At node 1,

$$-2 + \frac{V_1 - 0}{5} + \frac{V_1 - V_2}{j4} = 0 \quad (1)$$

$$(0.2 - j0.25)V_1 + j0.25V_2 = 2$$

At node 2,

$$\frac{V_2 - V_1}{j4} + \frac{V_2 - 0}{-j3} - 3\angle 45^\circ = 0 \quad (2)$$

$$j0.25V_1 + j0.08333V_2 = 2.121 + j2.121$$

In matrix form, (1) and (2) become

$$\begin{bmatrix} 0.2 - j0.25 & j0.25 \\ j0.25 & j0.08333 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2.121 + j2.121 \end{bmatrix}$$

Solving this using MATLAB, we get,

```
>> Y=[(0.2-0.25i),0.25i;0.25i,0.08333i]
```

Y =

$$\begin{array}{cc} 0.2000 - 0.2500i & 0 + 0.2500i \\ 0 + 0.2500i & 0 + 0.0833i \end{array}$$

```
>> I=[2;(2.121+2.121i)]
```

I =

2.0000
2.1210 + 2.1210i

>> V=inv(Y)*I

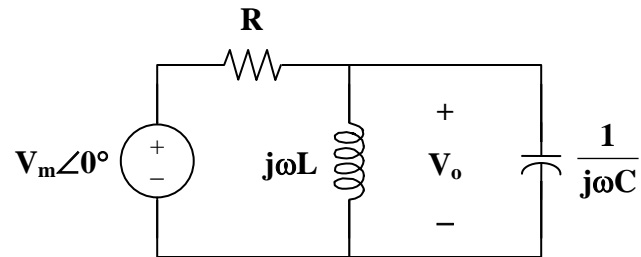
V =

5.2793 - 5.4190i
9.6145 - 9.1955i

$$V_s = V_1 - V_2 = -4.335 + j3.776 = \underline{\underline{5.749 \angle 138.94^\circ \text{ V}}}.$$

Chapter 10, Solution 20.

The circuit is converted to its frequency-domain equivalent circuit as shown below.



$$\text{Let } \mathbf{Z} = j\omega L \parallel \frac{1}{j\omega C} = \frac{\frac{L}{C}}{j\omega L + \frac{1}{j\omega C}} = \frac{j\omega L}{1 - \omega^2 LC}$$

$$\mathbf{V}_o = \frac{\mathbf{Z}}{\mathbf{R} + \mathbf{Z}} \mathbf{V}_m = \frac{\frac{j\omega L}{1 - \omega^2 LC}}{\mathbf{R} + \frac{j\omega L}{1 - \omega^2 LC}} \mathbf{V}_m = \frac{j\omega L}{\mathbf{R}(1 - \omega^2 LC) + j\omega L} \mathbf{V}_m$$

$$\mathbf{V}_o = \frac{\omega L \mathbf{V}_m}{\sqrt{\mathbf{R}^2 (1 - \omega^2 LC)^2 + \omega^2 L^2}} \angle \left(90^\circ - \tan^{-1} \frac{\omega L}{\mathbf{R}(1 - \omega^2 LC)} \right)$$

If $\mathbf{V}_o = A \angle \phi$, then

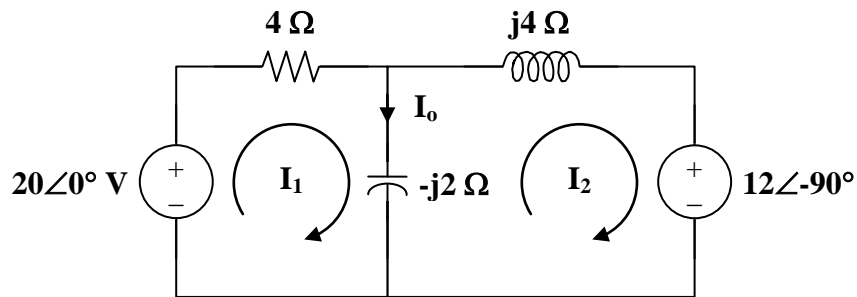
$$A = \frac{\omega L \mathbf{V}_m}{\sqrt{\mathbf{R}^2 (1 - \omega^2 LC)^2 + \omega^2 L^2}}$$

and
$$\phi = \underline{\underline{90^\circ - \tan^{-1} \frac{\omega L}{\mathbf{R}(1 - \omega^2 LC)}}}$$

Chapter 10, Solution 25.

$$\begin{aligned}\omega &= 2 \\ 10 \cos(2t) &\longrightarrow 20 \angle 0^\circ \\ 6 \sin(2t) &\longrightarrow 12 \angle -90^\circ = -j12 \\ 2 \text{ H} &\longrightarrow j\omega L = j4 \\ 0.25 \text{ F} &\longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(1/4)} = -j2\end{aligned}$$

The circuit is shown below.



For loop 1,

$$\begin{aligned}-20 + (4 - j2)\mathbf{I}_1 + j2\mathbf{I}_2 &= 0 \\ 10 &= (2 - j)\mathbf{I}_1 + j\mathbf{I}_2\end{aligned}\quad (1)$$

For loop 2,

$$\begin{aligned}j2\mathbf{I}_1 + (j4 - j2)\mathbf{I}_2 + (-j12) &= 0 \\ \mathbf{I}_1 + \mathbf{I}_2 &= 6\end{aligned}\quad (2)$$

In matrix form (1) and (2) become

$$\begin{bmatrix} 2 - j & j \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \end{bmatrix}$$

$$\Delta = 2(1 - j), \quad \Delta_1 = 10 - j6, \quad \Delta_2 = 2 - j6$$

$$\mathbf{I}_o = \mathbf{I}_1 - \mathbf{I}_2 = \frac{\Delta_1 - \Delta_2}{\Delta} = \frac{8}{2(1 - j)} = 2 + j2 = 2.828 \angle 45^\circ$$

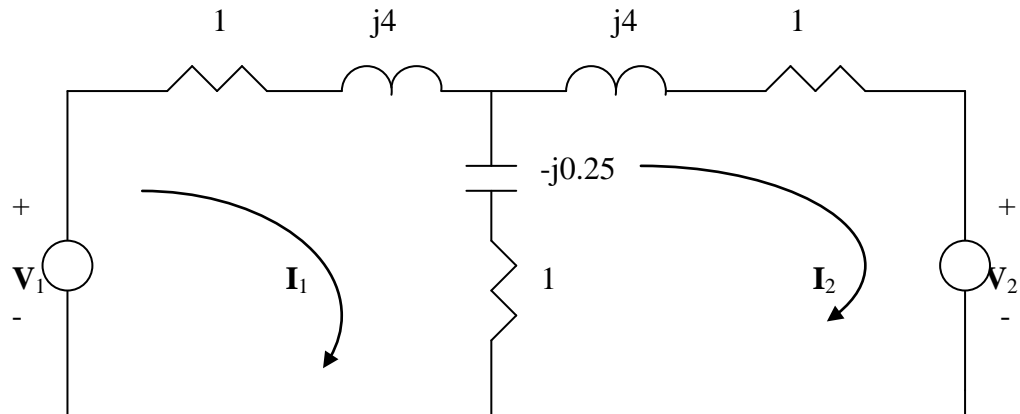
Therefore, $i_o(t) = \underline{\underline{2.828 \cos(2t + 45^\circ) \text{ A}}}$

Chapter 10, Solution 28.

$$1\text{H} \longrightarrow j\omega L = j4, \quad 1\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j1 \times 4} = -j0.25$$

The frequency-domain version of the circuit is shown below, where

$$V_1 = 10\angle 0^\circ, \quad V_2 = 20\angle -30^\circ.$$



$$V_1 = 10\angle 0^\circ, \quad V_2 = 20\angle -30^\circ$$

Applying mesh analysis,

$$10 = (2 + j3.75)I_1 - (1 - j0.25)I_2 \quad (1)$$

$$-20\angle -30^\circ = -(1 - j0.25)I_1 + (2 + j3.75)I_2 \quad (2)$$

From (1) and (2), we obtain

$$\begin{pmatrix} 10 \\ -17.32 + j10 \end{pmatrix} = \begin{pmatrix} 2 + j3.75 & -1 + j0.25 \\ -1 + j0.25 & 2 + j3.75 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

Solving this leads to

$$I_1 = 2.741\angle -41.07^\circ, \quad I_2 = 4.114\angle 92^\circ$$

Hence,

$$i_1(t) = \underline{2.741\cos(4t - 41.07^\circ)}\text{A}, \quad i_2(t) = \underline{4.114\cos(4t + 92^\circ)}\text{A}.$$