HW10

Tuesday, February 23, 2010
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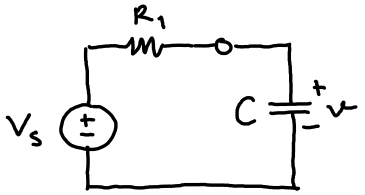
① Depending on the position of the switch (SW), we have two different circuits

(a) SW is at position 1



Note that the source V_s and the resistor R_1 are in the Thevenin equivalent circuit form already.

Therefore we have

$$\tau_1 = C \times R_{th} = C \times R_1$$
$$= 4\times10^{-6} \times 30\times10^{3}$$
$$= 12\times10^{-2} = 120 \text{ ms}$$
$$= 0.12 \text{ sec}$$

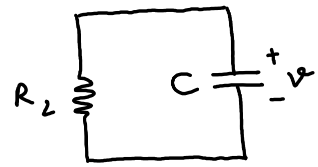
and

$$v(\infty) = V_s$$

When the circuit is in this setup, the voltage v across the capacitor is given by

$$v(t) = v(\infty) + (v(t_0) - v(\infty))\, e^{-(t-t_0)/\tau}$$
$$= V_s + (v(t_0) - V_s)\, e^{-(t-t_0)/R_1C}$$

(b) SW is at position 2



Here, $\tau_2 = R_2 \times C$
$$= 10\times10^{3} \times 4\times10^{-6}$$
$$= 4\times10^{-2} = 40 \text{ ms}$$
$$= 0.04 \text{ sec}$$

and

$$v(\infty) = 0$$

When the circuit is in this setup, the voltage v across the capacitor is given by

$$v(t) = \cancelto{0}{v(\infty)} + (v(t_0) - \cancelto{0}{v(\infty)})\, e^{-(t-t_0)/\tau}$$
$$= v(t_0)\, e^{-(t-t_0)/R_2C}$$

Note that SW changes its position every 25 ms.
25 ms is much less than $5\times\tau$ for both (a) and (b) above.
Hence, the voltage will not reach (or even come close) to its long-term (∞) value.

Case 1: $t < t_1 = 0$, SW is at position 1

Because SW has been in position 1 for a long time, we may assume that it has already reached $v(\infty)$ which is V_s from (a) above, that is

$$v(0^-) = V_s = 10\text{ V}$$

Because there can not be any voltage jump across

Because there can not be any voltage jump across the capacitor,

$$v(0) = v(0^-) = V_s = 10\ V.$$

case 2 : $t_1 \leq t < t_2$, sw is at position 2

$t_1 = 0$, $t_2 = 25\,ms$

Start time for this case is $t_0 = t_1 = 0$

$$v(t) = v(t_0)e^{-(t-t_0)/R_2C} = v(0)\,e^{-t/R_2C} = 10\,e^{-25t}$$

$$v(t_2^-) = 10\,e^{-25\times10^{-3}/R_2\times C} = 5.353\ V$$

case 3 : $t_2 \leq t < t_3$, sw is at position 1

Now this is the new t_0

$$v(t) = V_s + (v(t_2) - V_s)\,e^{-(t-t_2)/R_1C}$$

$$= 10 - 4.647\,e^{-8.333(t-t_2)}$$

$$v(t_3^-) = 10 - 4.647\,e^{-(t_3-t_2)/R_1C} = 10 - 4.647\,e^{-25\times10^{-3}/R_1C}$$

$$= 6.227\ V$$

Case 4 : $t_3 \leq t < t_4$, sw is at position 2

$$v(t) = v(t_3)\,e^{-(t-t_3)/R_2C}$$

$$= 6.227\,e^{-25(t-t_3)}$$

$$v(t_4^-) = 6.227\,e^{-25\times10^{-3}/R_2C} = 3.333\ V$$

Case 5 : $t_4 \leq t < t_5$, sw is at position 1

$$v(t) = V_s + (v(t_4) - V_s)\,e^{-(t-t_4)/R_1C}$$

$$= 10 - 6.667\,e^{-8.333(t-t_4)}$$

$$v(t_5) = 10 - 6.667\,e^{-25\times10^{-3}/R_1C}$$

$$= 4.587\ V$$

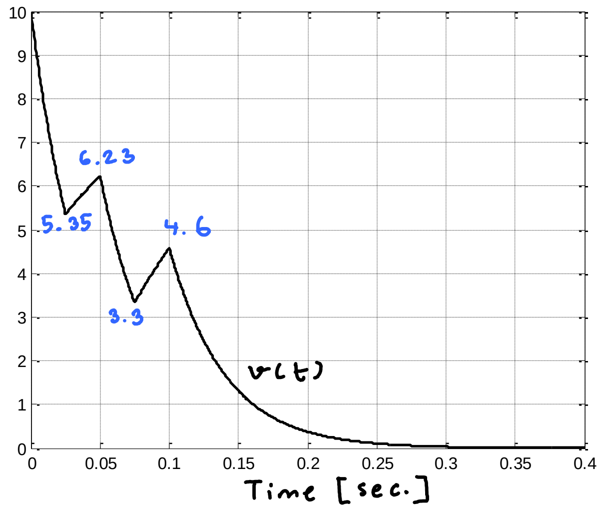
case 6 : $t > t_5$, sw is at position 2

$$v(t) = v(t_5)\,e^{-(t-t_5)/R_2C} = 4.587\,e^{-25(t-t_5)}$$

$$v(\infty) = 0$$

$$
v(t) = \begin{cases} 10\,e^{\cdots}, & 0 \le t < 25\,ms \\ 10 - 4.647\,e^{-8.333(t-25ms)}, & 25\,ms \le t < 50\,ms \\ 6.227\,e^{-25(t-50ms)}, & 50\,ms \le t < 75\,ms \\ 10 - 6.667\,e^{-8.333(t-75\,ms)}, & 75\,ms \le t < 100\,ms \\ 4.587\,e^{-25(t-100ms)}, & t \geq 100\,ms \end{cases}
$$

Here is the plot of $v(t)$:



2 a) $\boxed{8}$

b) $\boxed{500\pi \quad rad/s} \approx 1571 \quad rad/s$

c) $\omega = 2\pi f \Rightarrow f = \omega / 2\pi = 500\pi / 2\pi = \boxed{250 \; Hz}$

d) $8 \cos\left(500\pi \times 2 \times 10^{-3} - 25^\circ\right) = \boxed{-7.25}.$

3 a) $V = \boxed{21 \angle -15^\circ}$

b) $I = 8 \angle (70^\circ - 90^\circ + 180^\circ)$ sin → cos negative sign in the front

$= \boxed{8 \angle 160^\circ}$

c) $V = 120 \angle (-50^\circ - 90^\circ)$ sin → cos

$= \boxed{120 \angle -140^\circ}$

d) $I = 60 \angle (10^\circ - 180^\circ)$ negative sign

$= \boxed{60 \angle -170^\circ}$

e) $X = 10 \angle (75^\circ - 180^\circ)$ negative sign

$= \boxed{10 \angle -105^\circ}$

f) $X = 5\angle(\overset{\text{sin} \to \text{cos}}{-10^\circ - 90^\circ})$

$$= \boxed{5 \angle -100^\circ}$$

g) $x(t) = 4\cos 2t + 3\sin 2t = 4\cos 2t + 3\cos(2t - 90^\circ)$

$$= 4\,\mathrm{Re}\{e^{j2t}\} + 3\,\mathrm{Re}\{e^{j2t}(-j)\}$$

$$= \mathrm{Re}\{(\underbrace{4 - 3j}_{\to\, 5\angle -36.87^\circ})\, e^{j2t}\}$$

$$X = \boxed{5 \angle -36.87^\circ}$$

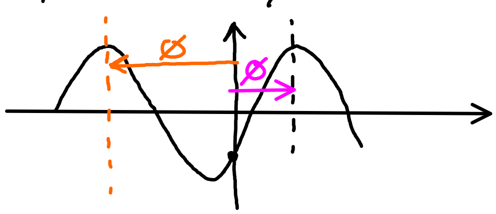
Alternatively, you may perform the addition in phasor form

$$X = 4\angle 0^\circ + 3\angle -90^\circ$$

$$= 4 + (-3j) = 4 - 3j$$

$$= 5 \angle -36.87^\circ$$

(4) First we observe that the wave form is the same as cosine function shifted to the right by ϕ where ϕ is between 90° and 180°



Shifting to the right means ϕ is negative.
So,

$$-180^\circ < \phi < -90^\circ$$

Equivalently, the graph is also the cosine function shifted to the left by ϕ where

$$180^\circ < \phi < 270^\circ$$

Now,
from the general form of sinusoidal waveform

$$x(t) = A\cos(\omega t + \phi)$$

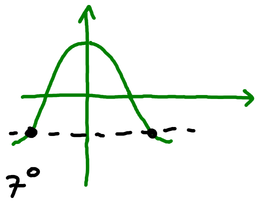
From the plot, we have $A = 4$.

$$x(0) = 4\cos(\phi)$$

$\shortparallel$

-3.356

$$\cos\phi = -\frac{3.356}{4}$$
$$\approx -0.839$$



$$\phi = 147^\circ \text{ and } -147^\circ$$

Because ϕ must be between -180° and -90°, we know that $\phi = -147^\circ$.

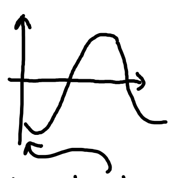
Equivalently, $\phi = -147^\circ + 360^\circ = 213^\circ$

Therefore,

$$\boxed{\vec{X} = 4 \angle -147^\circ}$$

Note that $\phi = 147^\circ$ will give different graph.

Try it! You will get



which start at the wrong position.