ECS 303 - Part 1D
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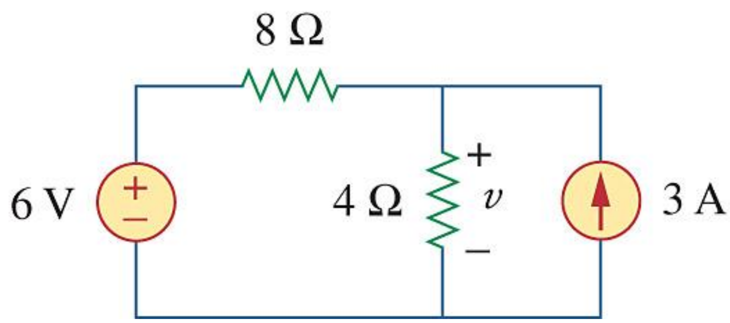
4.2. Superposition

Superposition technique = A way to determine currents and voltages in a circuit that has multiple independent sources by considering the contribution of one source at a time and then add them up.

Ex.



The **superposition principle** states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

However, to apply the superposition principle, we must keep two things in mind.

1. We consider one independent source at a time while all other independent source are *turned off*.[1]
   - Replace other independent voltage sources by 0 V (or short circuits)
   - Replace other independent current sources by 0 A (or open circuits)
2. Dependent sources are left intact because they are controlled by circuit variable.

Steps to Apply Superposition Principles:

S1: Turn off all independent sources except one source. Find the output due to that active source.

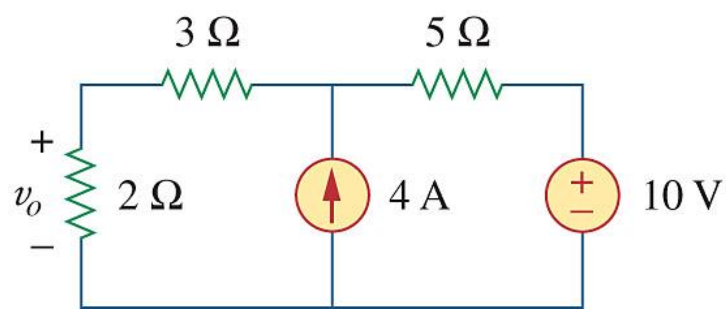
S2: Repeat S1 for each of the other independent sources.

[1] Other terms such as killed, made inactive, deadened, or set equal to zero are often used to convey the same idea.

S3: Find the total contribution by adding algebraically all the contributions due to the independent sources.

Keep in mind that superposition is based on linearity. Hence, we cannot find the total power from the power due to each source, because the power absorbed by a resistor depends on the square of the voltage or current and hence it is not linear (e.g. because $5^2 \neq 1^2 + 4^2$).

Ex.

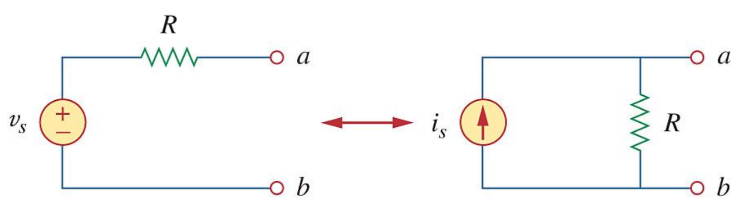


Remark: Superposition helps reduce a complex circuit to simpler circuits through replacement of voltage sources by short circuits and of current sources by open circuits.

4.3. Source Transformation

We have noticed that series-parallel combination helps simplify circuits. The simplification is done by replacing replacing one part of a circuit by its equivalence.[2] Source transformation is another tool for simplifying circuits.

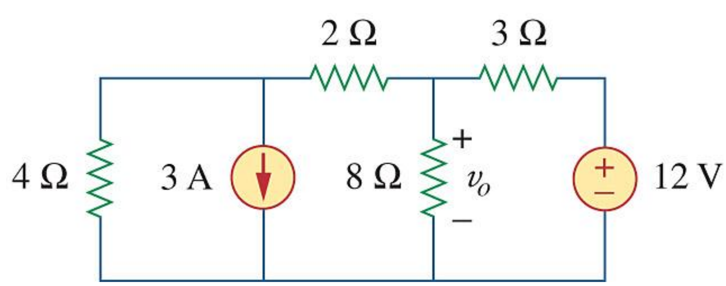
A **source transformation** is the process of replacing a voltage source in series with a resistor R by a current source in parallel with a resistor R or vice versa.



Notice that when terminals $a-b$ are short-circuited, the short-circuit current flowing from a to b is $i_{sc} = v_s/R$ in the circuit on the left-hand side and $i_{sc} = i_s$ for the circuit on the righthand side. Thus, $v_s/R = i_s$ in order for the two circuits to be equivalent. Hence, source transformation requires that

$$v_s = i_s R \quad \text{or} \quad i_s = \frac{v_s}{R}. \tag{4.1}$$

Ex. Use source transformation to find v_0 in the following circuit:



Remark: From (4.1), an ideal voltage source with $R = 0$ cannot be replaced by a finite current source. Similarly, an ideal current source with $R = \infty$ cannot be replaced by a finite voltage source.

[2] Recall that an equivalent circuit is one whose $v - i$ characteristics are identical with the original circuit.

4.4. Thevenin's Theorem

It often occurs in practice that a particular element in a circuit is variable (usually called the **load**) while other elements are fixed.

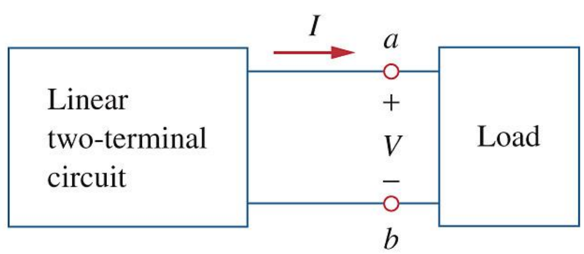
- As a typical example, a household outlet terminal may be connected to different appliances constituting a variable load.

Each time the variable element is changed, the entire circuit has to be analyzed all over again. To avoid this problem, Thevenins theorem provides a technique by which the fixed part of the circuit is replaced by an equivalent circuit.

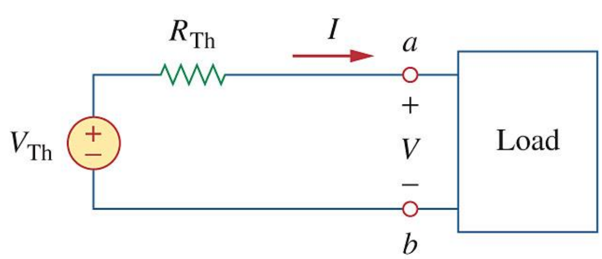
Thevenin's Theorem is an important method to simplify a complicated circuit to a very simple circuit. It states that a circuit can be replaced by an equivalent circuit consisting of an independent voltage source V_{TH} in series with a resistor R_{TH}, where

V_{TH}: the open circuit voltage at the terminal.

R_{TH}: the equivalent resistance at the terminals when the independent sources are turned off.



(a)

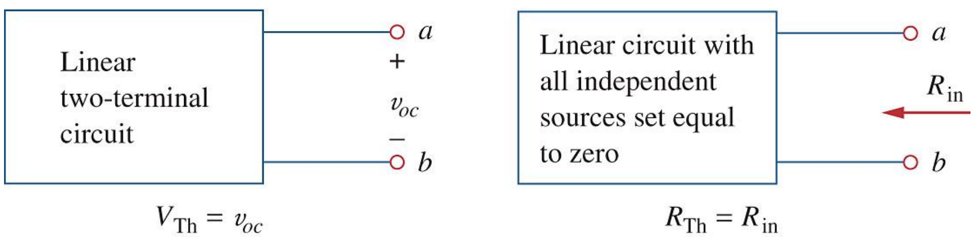


(b)

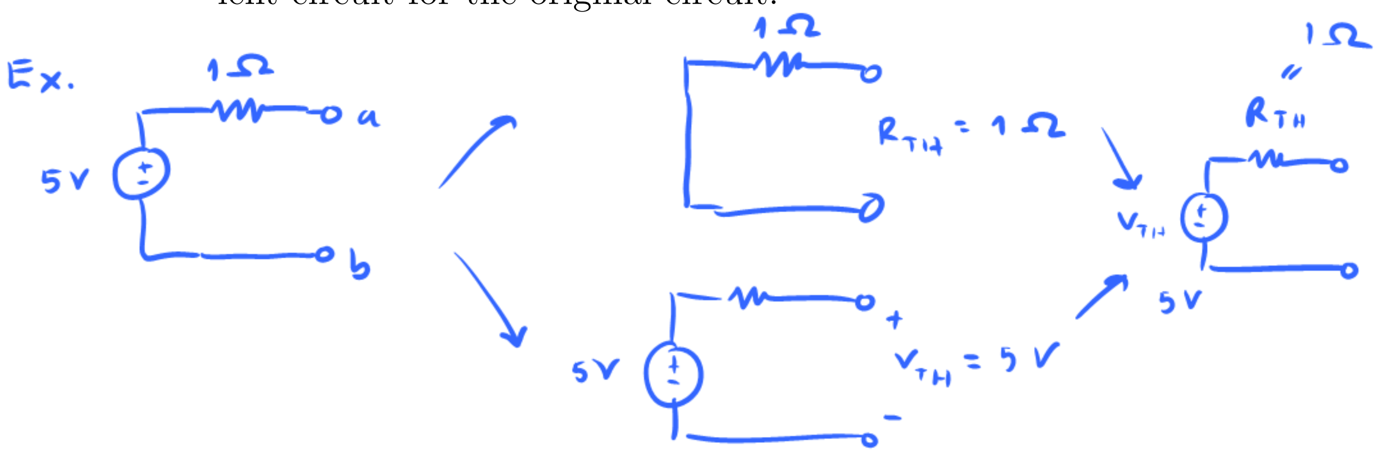
Steps to Apply Thevenin's theorem. (Case I: No dependent source)

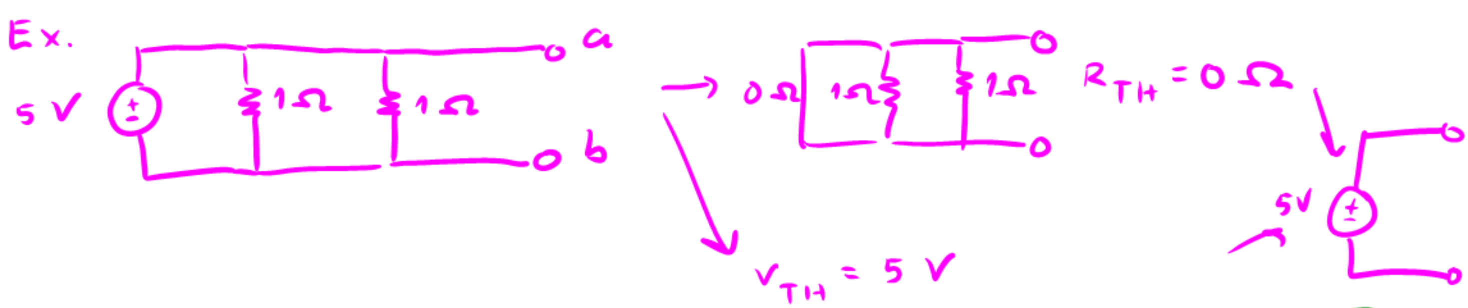
S1: Find R_{TH}: Turn off all independent sources. R_{TH} is the input resistance of the network looking between terminals a and b.

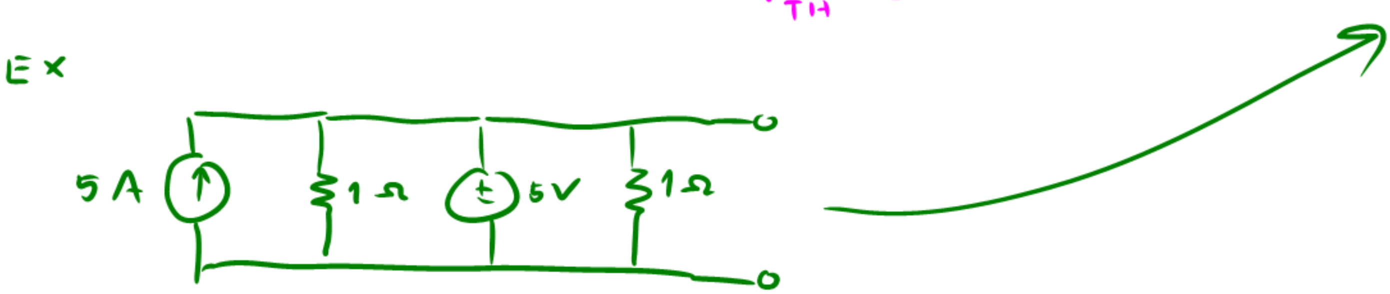
S2: Find V_{TH}: Open the two terminals (remove the load) which you want to find the Thevenin equivalent circuit. V_{TH} is the open-circuit voltage across the terminals.



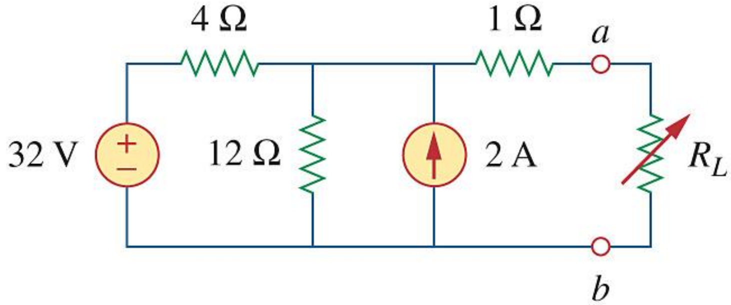
S3: Connect V_{TH} and R_{TH} in series to produce the Thevenin equivalent circuit for the original circuit.



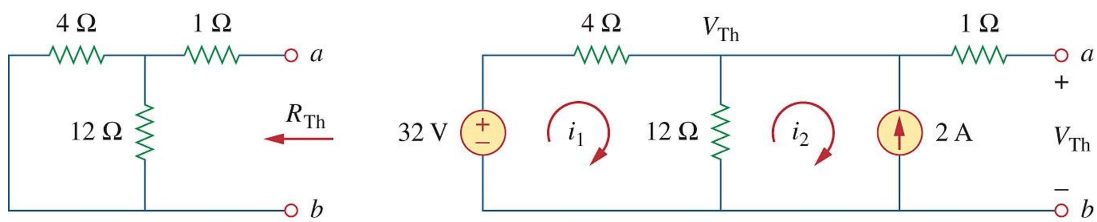




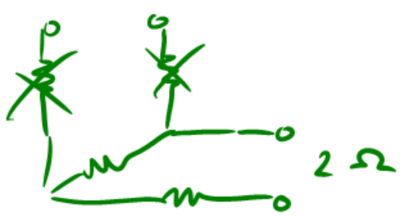
Ex. Find the Thevenin equivalent circuit of the circuit shown below, to the left of the terminals a-b. Then find the current through $R_L = 6$, 16, and 36 .

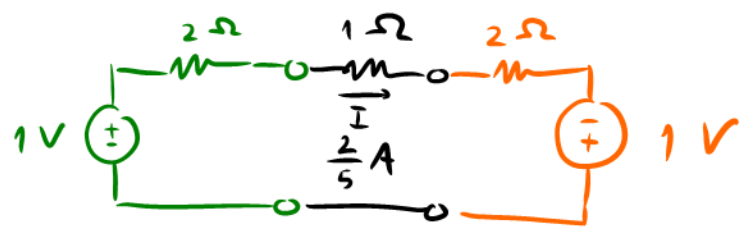


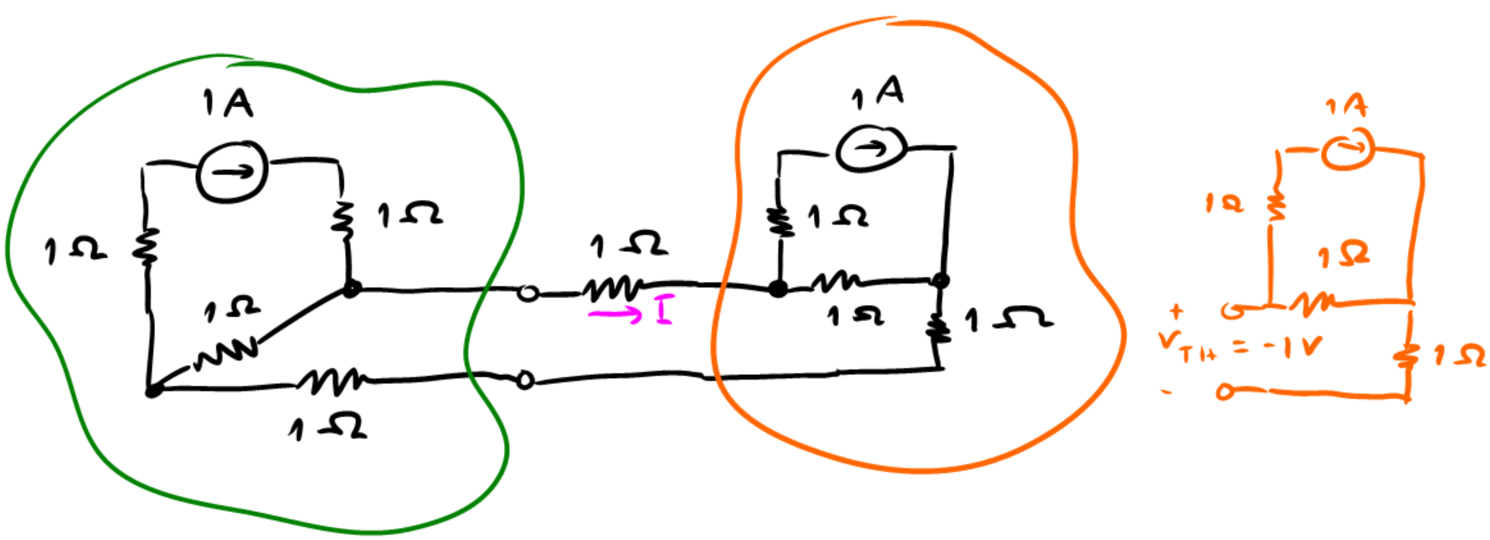
Solution:



(a) (b)







Steps to Apply Thevenin's theorem.(Case II: with dependent sources)

S1: Find R_{TH}:

S1.1 Turn off all independent sources (but leave the dependent sources on).

S1.2 Apply a voltage source v_o at terminals a and b, determine the resulting current i_o, then

$$R_{TH} = \frac{v_o}{i_o}$$

Note that: We usually set $v_o = 1$ V.

Or, equivalently,

S1.2 Apply a current source i_o at terminal a and b, find v_o, then $R_{TH} = \frac{v_o}{i_o}$

S2: Find V_{TH}, as the open-circuit voltage across the terminals.

S3: Connect R_{TH} and V_{TH} in series.

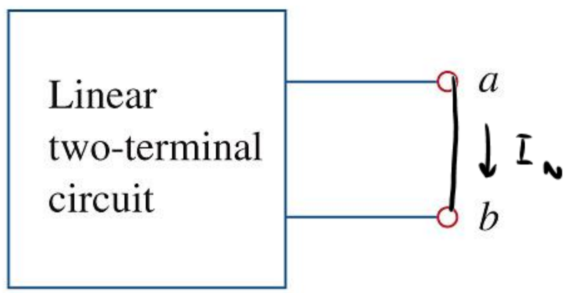
Remark: It often occurs that R_{TH} takes a negative value. In this case, the negative resistance implies that the circuit is supplying power. This is possible in a circuit with dependent sources.

4.5. Norton's Theorem

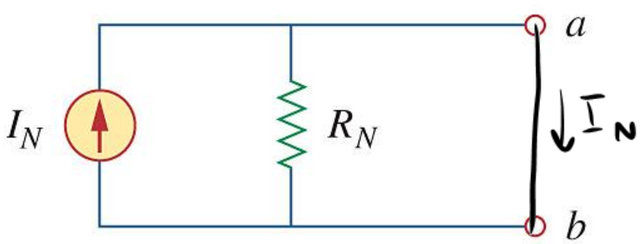
Norton's Theorem gives an alternative equivalent circuit to Thevenin's Theorem.

Norton's Theorem: A circuit can be replaced by an equivalent circuit consisting of a **current source** I_N in **parallel** with a **resistor** R_N, where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.

Note: $\boxed{\mathrm{R}_N = R_{TH} \text{ and } I_N = \frac{V_{TH}}{R_{TH}}.}$ These relations are easily seen via source transformation.[3]



(a)



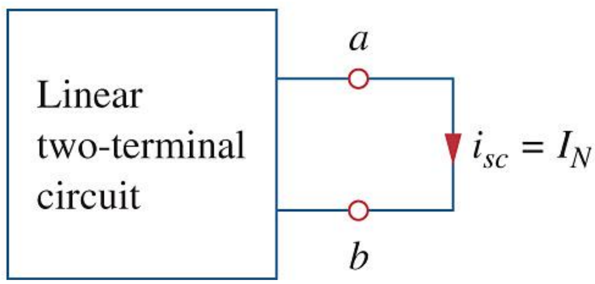
(b)

[3]For this reason, source transformation is often called Thevenin-Norton transformation.

Steps to Apply Norton's Theorem

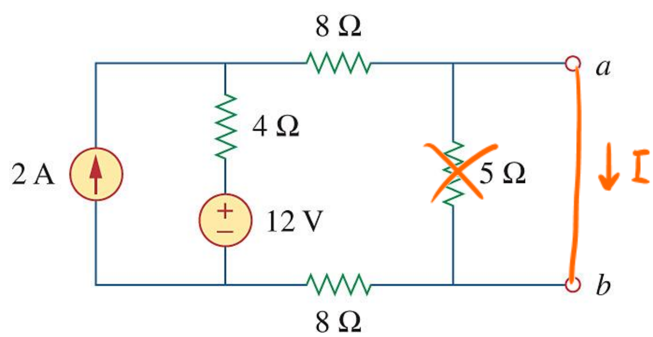
S1: Find R_N (in the same way we find R_{TH}).

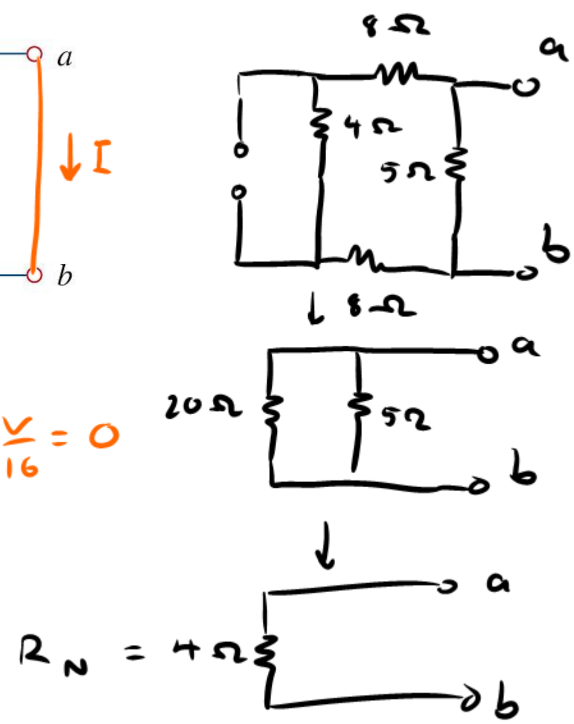
S2: Find I_N: Short circuit terminals a to b. I_N is the current passing through a and b.

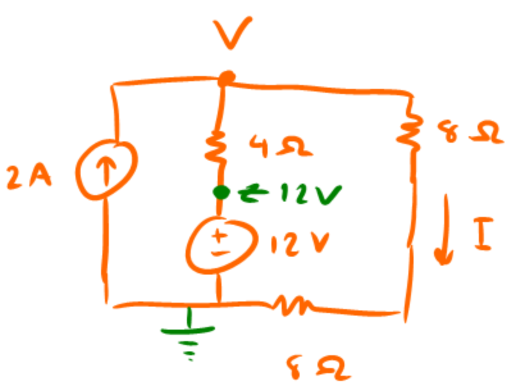


S3: Connect I_N and R_N in parallel.

Ex. Find the Norton equivalent circuit of the circuit in the following figure at terminals a-b.



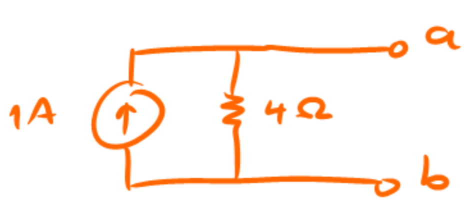




$$-2 + \frac{V-12}{4} + \frac{V}{16} = 0$$

$$V = 16\text{ V}$$

$$I = \frac{16}{8+8} = 1\text{ A}$$



4.6. Maximum Power Transfer

In many practical situations, a circuit is designed to provide power to a load. In areas such as communications, it is desirable to maximize the power delivered to a load. We now address the problem of delivering the maximum power to a load when given a system with known internal losses.

The Thevenin and Norton models imply that some of the power generated by the source will necessarily be dissipated by the internal circuits within the source.

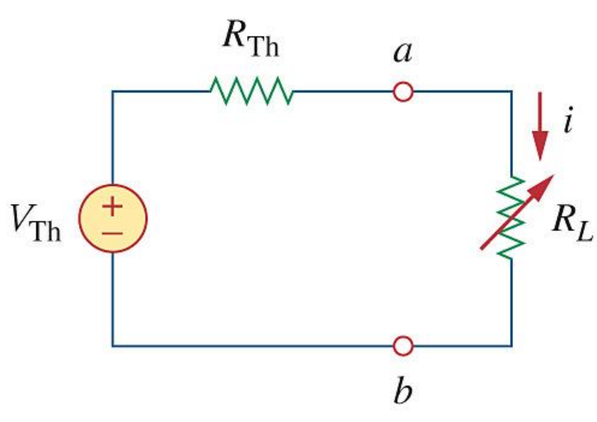
Questions:

(a) How much power can be transferred to the load under the most ideal conditions?

(b) What is the value of the load resistance that will absorb the maximum power from the source?

If the entire circuit is replaced by its Thevenin equivalent except for the load, as shown below, the power delivered to the load resistor R_L is

$$p = i^2 R_L \quad \text{where } i = \frac{V_{th}}{R_{th} + R_L}$$



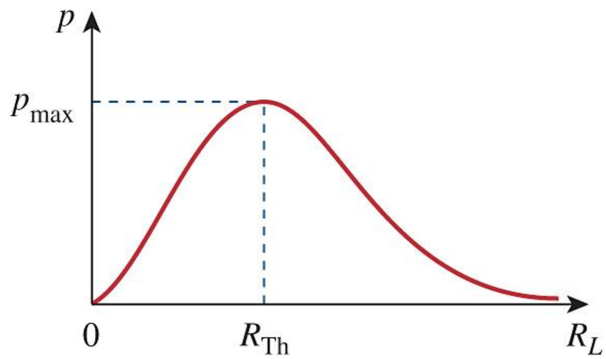
The derivative of p with respect to R_L is given by

$$\begin{aligned}
\frac{dp}{dR_L} &= 2i\frac{di}{dR_L}R_L + i^2 \\
&= 2\frac{V_{th}}{R_{th}+R_L}\left(-\frac{V_{th}}{(R_{th}+R_L)^2}\right) + \left(\frac{V_{th}}{R_{th}+R_L}\right)^2 \\
&= \left(\frac{V_{th}}{R_{th}+R_L}\right)^2\left(-\frac{2R_L}{R_{th}+R_L}+1\right).
\end{aligned}$$

We then set this derivative equal to zero and get

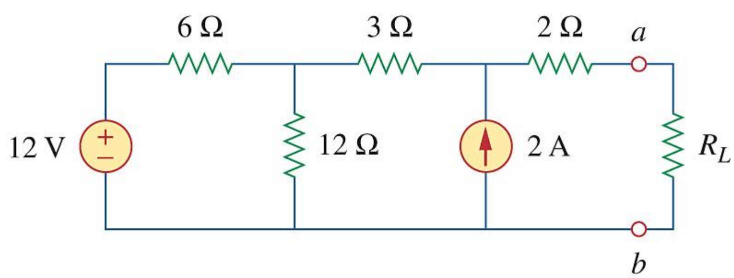
$$R_L = R_{TH}$$

showing that the maximum power transfer takes place when the load resistance R_L equals the Thevenin resistance R_{Th}.



$$P_{MAX} = i^2 R_L = \left(\frac{V_{TH}}{2R_{TH}}\right)^2 R_{TH} = \frac{V_{TH}^2}{4R_{TH}}$$

Ex. Find the value of R_L for maximum power transfer in the circuit below. Find the maximum power.



Ex.