ECS 303 - Part 1C
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CHAPTER 4

Circuit Theorems

The growth in areas of application of electrical circuits has led to an evolution from simple to complex circuits. To handle such complexity, engineers over the years have developed theorems to simplify circuit analysis. These theorems (Thevenin's and Norton's theorems) are applicable to *linear* circuits which are composed of resistors, voltage and current sources.

4.1. Linearity Property

A **linear element** (e.g., resistor) with input i and output v has the following properties:

- Homogeneous (Scaling): If i is multiplied by a constant k, then the output is multiplied by k.

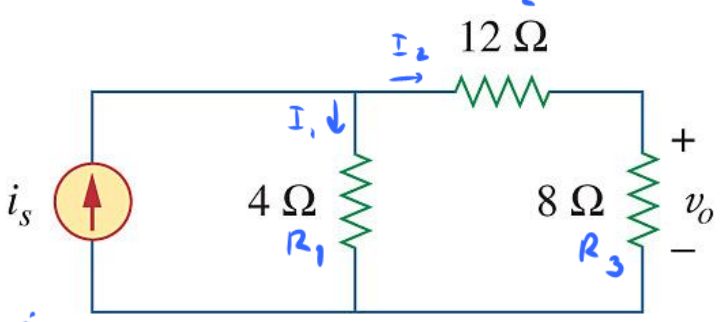
$$iR = v \Rightarrow (ki)R = kv$$

- Additive: If the inputs are summed then the output are summed.

$$i_1 R = v_1,\ i_2 R = v_2 \Rightarrow (i_1 + i_2)R = v_1 + v_2$$

A **linear circuit** is a circuit whose output is linearly related (or directly proportional) to its input.

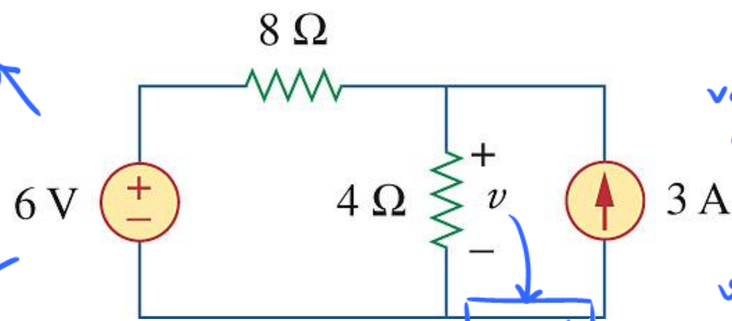
Ex.



4.2. Superposition

Superposition technique = A way to determine currents and voltages in a circuit that has multiple independent sources by considering the contribution of one source at a time and then add them up.

Ex.



If you have many sources: 1 voltage source V_s, 1 current source I_s

voltage/current output $= k_1 V_s + k_2 I_s$

$v = \frac{4}{4+8} \times 6 = 2$ V

$v = 8$ V

$8 \parallel 4 = \frac{8 \times 4}{8+4} = \frac{32}{12} = \frac{8}{3}$

$v = k_1 \times V_s + k_2 \times I_s$

$v = k_1 V_s$ $\quad v = k_2 I_s$

10 V

The **superposition principle** states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

However, to apply the superposition principle, we must keep two things in mind.

1. We consider one independent source at a time while all other independent source are *turned off*.[1]
   - Replace other independent voltage sources by 0 V (or short circuits)
   - Replace other independent current sources by 0 A (or open circuits)
2. Dependent sources are left intact because they are controlled by circuit variable.

Steps to Apply Superposition Principles:

S1: Turn off all independent sources except one source. Find the output due to that active source.

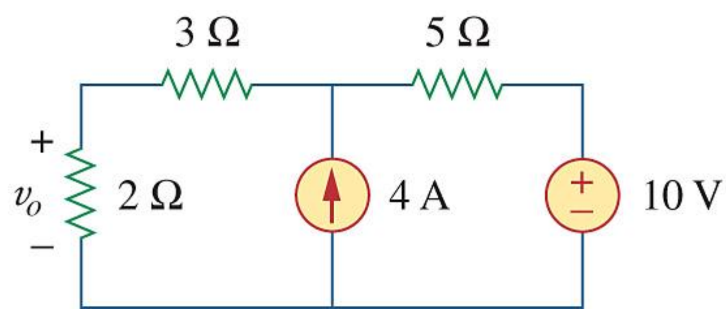
S2: Repeat S1 for each of the other independent sources.

[1] Other terms such as killed, made inactive, deadened, or set equal to zero are often used to convey the same idea.

S3: Find the total contribution by adding algebraically all the contributions due to the independent sources.

Keep in mind that superposition is based on linearity. Hence, we cannot find the total power from the power due to each source, because the power absorbed by a resistor depends on the square of the voltage or current and hence it is not linear (e.g. because $5^2 \neq 1^2 + 4^2$).

Ex.

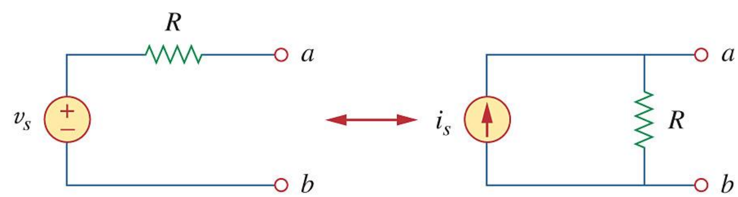


4.3. Source Transformation

We have noticed that series-parallel combination helps simplify circuits. The simplification is done by replacing replacing one part of a circuit by its equivalence.[2] Source transformation is another tool for simplifying circuits.

A **source transformation** is the process of replacing a voltage source in series with a resistor R by a current source in parallel with a resistor R or vice versa.

Ex.



Notice that when terminals $a - b$ are short-circuited, the short-circuit current flowing from a to b is $i_{sc} = v_s/R$ in the circuit on the left-hand side and $i_{sc} = i_s$ for the circuit on the righthand side. Thus, $v_s/R = i_s$ in order for the two circuits to be equivalent. Hence, source transformation requires that

$$v_s = i_s R \quad \text{or} \quad i_s = \frac{v_s}{R}.$$

[2]Recall that an equivalent circuit is one whose $v - i$ characteristics are identical with the original circuit.