ECS 303 - Part 1B
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CHAPTER 3

Methods of Analysis

Here we apply the fundamental laws of circuit theory (Ohm's Law & Kirchhoff's Laws) to develop two powerful techniques for circuit analysis.
1. Nodal Analysis (based on KCL)
2. Mesh Analysis (based on KVL)

This is the *most important* chapter for our course.

3.1. Nodal Analysis

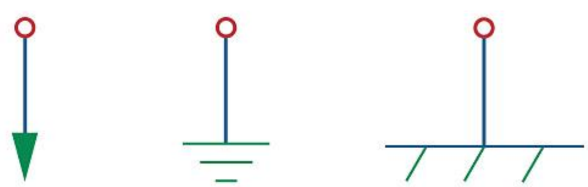
Analyzing circuit using **node voltages** as the circuit variables.

Steps to Determine Node Voltages:

Step 0: Determine the number of nodes n.

Step 1: Select a node as a reference node (ground node). Assign voltages v_1, v_2, $\cdots$, v_{n-1} to the remaining $n-1$ nodes. The voltage are referenced with respect to the reference node.

- The ground node is assumed to have 0 potential.



Step 2: Apply KCL to each of the $n-1$ nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.

Step 3: Solve the resulting *simultaneous equations* to obtain the unknown node voltages.

Remark:

(a) Current flows from a higher potential to a lower potential in a resistor.

(b) If a voltage source is connected between the reference node and a nonreference node, we simply set the voltage at the nonreference node equal to the voltage of the voltage source.

(c) Multiple methods to solve the simultaneous equations in Step 3.

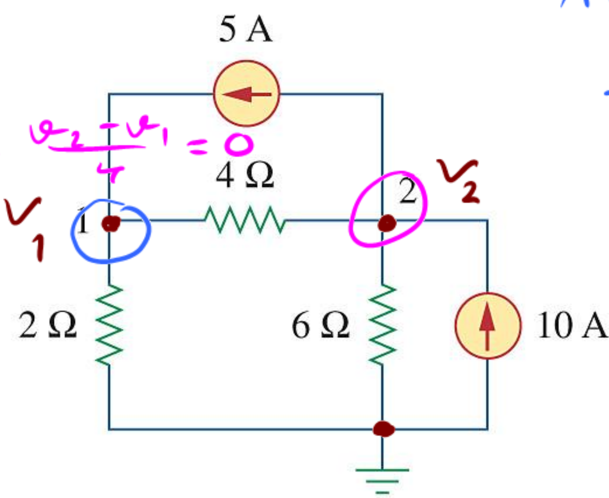
- Method 1: Elimination technique (good for a few variables)
- Method 2: Write in term of matrix and vectors (write $Ax = b$), then use Cramer's rule.
- Method 3: Use computer (e.g., Matlab) to find A^{-1} and $x = A^{-1}b$

Ex.

At node ② : KCL

$5 + (-10) + \frac{v_2 - 0}{6} + \frac{v_2 - v_1}{4} = 0$

$-\frac{1}{4}v_1 + \left(\frac{1}{6}+\frac{1}{4}\right)v_2 = 5$



At node ① : KCL →

$-5 + \frac{v_1}{2} + \frac{v_1 - v_2}{4} = 0$

$\left(\frac{1}{2}+\frac{1}{4}\right)v_1 - \frac{1}{4}v_2 = 5$

$v_1 = \frac{40}{3}, \; v_2 = 20.$

Ex.

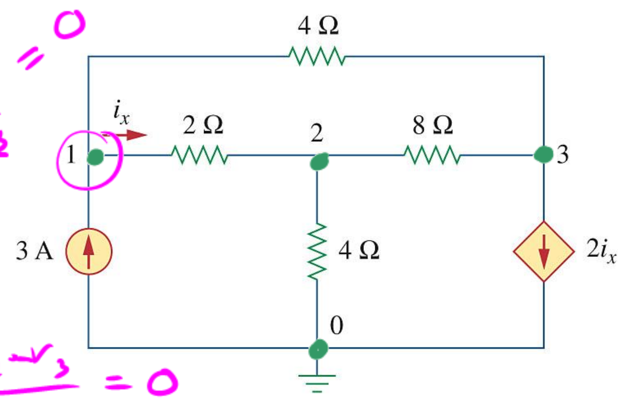
$i_x = \frac{v_1 - v_2}{2}$

Node 1:

$-3 + \frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4} = 0$

Node 2:

$\frac{v_2 - v_1}{2} + \frac{v_2 - 0}{4} + \frac{v_2 - v_3}{8} = 0$



Node 3:

$\frac{v_3 - v_1}{4} + \frac{v_3 - v_2}{8} + 2i_x = 0$ ← $v_1 - v_2$

Special Case: If there is a voltage source connected between two non-reference nodes, the two nonreference nodes form a **supernode**. We apply both KCL and KVL to determine the node voltages.

$v_1 = 4.8$ V

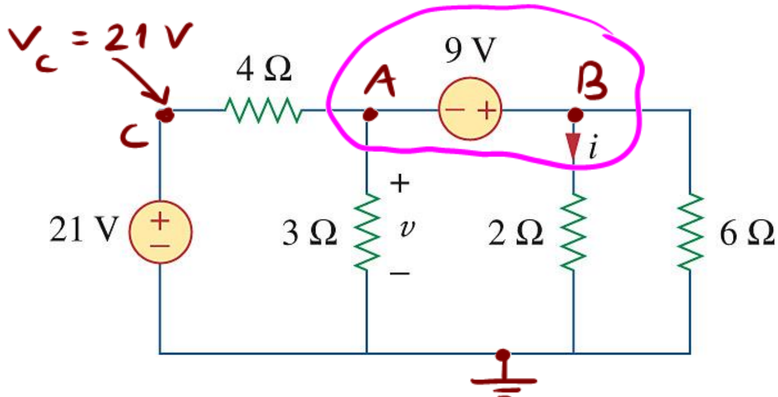
$v_2 = 2.4$ V

$v_3 = -2.4$ V

KCL at supernode: $\frac{V_A - 21}{4} + \frac{V_A}{3} + \frac{V_B}{2} + \frac{V_B}{6} = 0$

$v_B - v_A = 9$

Ex.



$v_A = -0.6\ V$

$v_B = 8.4\ V$

Remarks: Note the following properties of a supernode:

(a) The voltage source inside the supernode provides a constraint equation needed to solve for the node voltages.

(b) A supernode has no voltage of its own.

(c) We can have more than two nodes forming a single supernode.

(d) The supernodes are treated differently because nodal analysis requires knowing the current through each element. However, there is no way of knowing the current through a voltage source in advance.

3.2. Mesh Analysis

Mesh analysis provides another general procedure for analyzing circuits, using **mesh currents** as the circuit variables.

Mesh is a loop which does not contain any other loop within it.

Steps to Determine Mesh Currents:

Step 0: Determine the number of meshes n.

Step 1: Assign mesh current $i_1, i_2, \cdots, i_n$, to the n meshes.

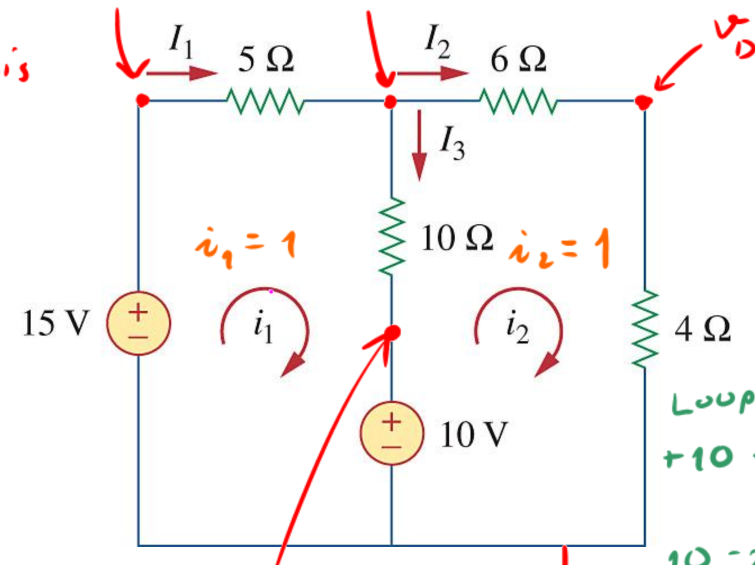
- The direction of the mesh current is arbitrary–(clockwise or counterclockwise)–and does not affect the validity of the solution.
- For convenience, we define currents flow in the clockwise (CW) direction.

Step 2: From the current direction in each mesh, define the voltage drop polarities.

Step 3: Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh current.

Step 4: Solve the resulting n simultaneous equations to get the mesh current.

Ex.



Use Nodal Analysis

Node D:

$$\frac{v_D - v_C}{6} + \frac{v_D - 0}{4} = 0$$

$$\left(\frac{1}{6} + \frac{1}{4}\right) v_D = \frac{v_C}{6}$$

$$\left(\frac{4+6}{4\times 6}\right) v_D = \frac{v_C}{6}$$

$$v_D = v_C \times \frac{4}{10} = \frac{2}{5} v_C$$

Node C:

$$\frac{v_C - 15}{5} + \frac{v_C - 10}{10} + \frac{v_C - v_D}{6} = 0$$

$$\frac{7}{15}\left(\frac{1}{5} + \frac{1}{10} + \frac{1}{6}\right) v_C - \frac{1}{6} v_D = 4$$

$$\frac{7}{15} v_C - \frac{1}{6} \times \frac{2}{5} v_C = 4$$

$$\frac{6}{15} v_C = 4$$

$$v_C = \frac{4 \times 15}{6} = 10$$

$$v_D = \frac{2}{5} \times 10 = 4$$

Use mesh analysis

Loop 1:

$$+15 - 5 i_1 - 10 - (i_1 - i_2) 10 = 0$$

$$5 = 15 i_1 - 10 i_2$$

$$3 i_1 - 2 i_2 = 1$$

Loop 2:

$$+10 - (i_2 - i_1) \times 10 - i_2 \times 6 - i_2 \times 4 = 0$$

$$10 - 20 i_2 + 10 i_1 = 0$$

$$i_1 - 2 i_2 = -1$$

Remarks:

(a) Nodal analysis applies KCL to find unknown voltages in a given circuit, while mesh analysis applies KVL to find unknown currents.

(b) Using mesh currents instead of element currents as circuit variables is convenient and reduces the number of equations that must be solved simultaneously.

(c) Mesh analysis is not quite as general as nodal analysis because it is only applicable to a circuit that is *planar*.

- A planar circuit is one that can be drawn in a plane with no branches crossing one another; otherwise it is nonplanar.

3.3. Nodal Versus Mesh Analysis

You should be familiar with both methods. However, given a network to be analyzed, how do we know which method is better or more efficient?

Suggestion: Choose the method that results in smaller number of variables or equations.

- A circuit with fewer nodes than meshes is better analyzed using nodal analysis, while a circuit with fewer meshes than nodes is better analyzed using mesh analysis.

You can also use one method to check your results of the other method.