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<b>EXPERIMENT</b>	: 05 RC Circuit and Resonant RLC Circuit

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## I. OBJECTIVES

1. To investigate RC circuit with voltage step input.
2. To determine the resonant frequency and frequency response of a series RLC circuit.

## II. BASIC INFORMATION

### II.1 RC Circuit with Voltage Step Input

1. An RC circuit with voltage step input is shown in Figure 5-1-1. From KCL, we have

$$C \frac{d}{dt} v_{out}(t) + \frac{v_{out}(t) - v_{in}(t)}{R} = 0.$$

If the input voltage  $v_{in}(t)$  is fixed at a particular value  $V_S$  from time  $t_1$  to  $t_2$ , then the output voltage  $v_{out}(t)$  during this time interval is given by

$$v_{out}(t) = V_S + (v_{out}(t_1) - V_S) e^{-\frac{t-t_1}{\tau}}, \quad t_1 \leq t \leq t_2, \quad (1)$$

where  $v_{out}(t_1)$  is the initial output voltage (for this interval which starts at time  $t_1$ ) and  $\tau = RC$  is the time constant.

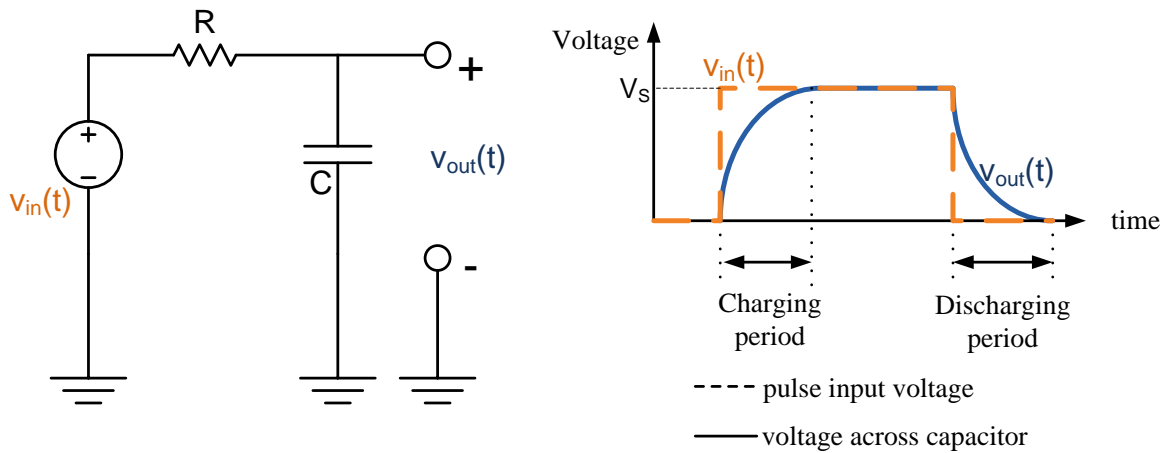
2. For simplicity, assume  $t_1 = 0$ . Then, the output voltage is given by

$$v_{out}(t) = V_S + (v_{out}(0) - V_S) e^{-\frac{t}{\tau}}.$$

Further assume that  $v_{out}(0) = 0$ . Then

$$v_{out}(t) = V_S - V_S e^{-\frac{t}{\tau}} = V_S \left( 1 - e^{-\frac{t}{\tau}} \right).$$

Therefore, the voltage across the capacitor increases from 0 to  $V_S$  and we say that the capacitor is **charging**. Note that in one time constant ( $t = \tau$ ), the capacitor is charged to approximately  $V_S(1 - e^{-1}) \approx 0.63V_S$ , i.e. the voltage across the capacitor becomes approximately 63 percent of  $V_S$ .



**Figure 5-1-1:** Pulse input voltage and the voltage across a capacitor.

3. In Part A of this experiment, the input voltage  $v_{in}(t)$  is a square-wave;  $v_{in}(t)$  will be 0 or  $V_S$ .
4. When  $v_{in}(t) \equiv 0$  from time  $t_1$  to  $t_2$ , the solution (1) above gives

$$v_{out}(t) = 0 + (v_{out}(t_1) - 0)e^{-\frac{t-t_1}{\tau}} = v_{out}(t_1)e^{-\frac{t-t_1}{\tau}}, \quad t_1 \leq t \leq t_2,$$

For simplicity, assume that  $t_1 = 0$  and that the initial output voltage  $v_{out}(t_1) = V_0$ . Then,

$$v_{out}(t) = V_0 e^{-\frac{t}{\tau}}.$$

Therefore, the voltage across the capacitor decreases from  $V_0$  to 0 and we say that the capacitor is **discharging**. In one time constant ( $t = \tau$ ), the capacitor is discharged to approximately  $V_0 e^{-\frac{\tau}{\tau}} = \frac{V_0}{e} \approx 0.37V_0$ , or 37 % of the initial voltage  $V_0$ .

5. **Half Life.** The half life  $t_{half}$  is the time  $t$  that is needed for the output voltage of a discharging capacitor to decrease from  $V_0$  to  $V_0/2$ . At such time,

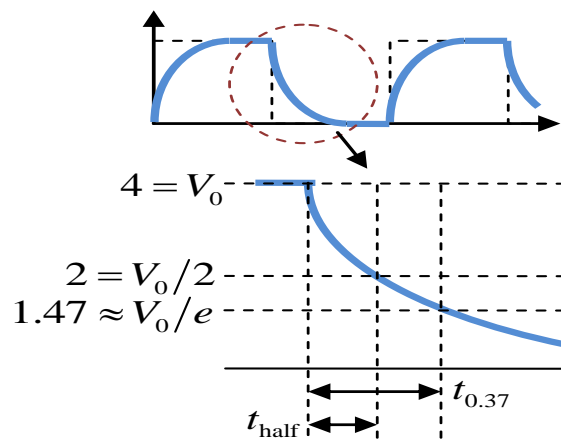
$$\frac{V_0}{2} = V_0 e^{-\frac{t}{\tau}}.$$

Solving for  $t$  in the above equation gives

$$t_{half} = \tau \ln 2.$$

6. In part A of this experiment, the value of  $\tau$  are found using three different methods (see Figure 5-1-2):

- 1) Measure  $t_{0.37}$ . Then,  $\tau = t_{0.37}$ .
- 2) Measure  $t_{half}$ . Then, calculate  $\tau = t_{half} / \ln 2$ .
- 3) Measure R and C. Then, calculate  $\tau = RC$ .



**Figure 5-1-2:**  $\tau$  measurement.

## II.2 Series RLC Circuit

1. Resistors limit the amount of current in dc as well as in ac circuits. In addition to resistors, reactive components, such as inductors and capacitors, impede currents in ac circuits.
2. **Inductive impedance** of a coil is given by

$$Z_L = j\omega L = j2\pi fL = jX_L \text{ ohms,}$$

where  $j = \sqrt{-1}$ ,  $L$  is the inductance in henries,  $f$  is the frequency of the signal in the circuit, and  $X_L = \text{Im}\{Z_L\} = \omega L = 2\pi fL$  is the **inductive reactance**. The ac voltage across a coil,  $V_L$  is equal to the product of the alternating current  $I$  in the coil and the inductive impedance  $Z_L$  of the coil; that is  $V_L = IZ_L$ .

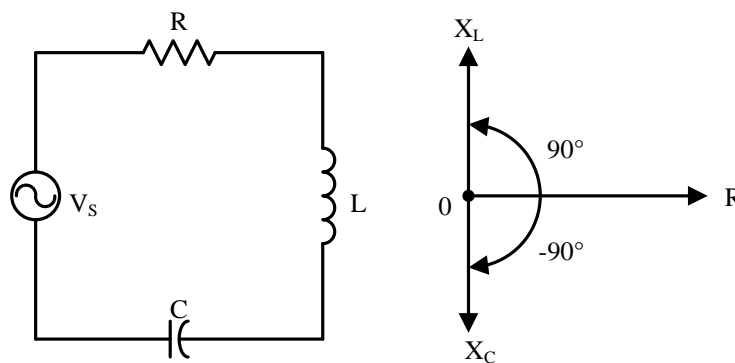
3. The capacitive impedance is given by

$$Z_C = 1/(j\omega C) = 1/(j2\pi fC) = -j/(2\pi fC) = -jX_C \text{ ohms,}$$

where  $X_C = 1/(\omega C) = 1/(2\pi fC)$  is the **capacitive reactance**. Note that  $X_C$  is inversely proportional to the frequency and the capacitance.

4. A series RLC circuit and its impedance phasor diagram is shown in Figure 5-2. The impedance of the series RLC circuit is

$$Z = R + Z_L + Z_C = R + j(X_L - X_C) = R + j\left(\omega L - \frac{1}{\omega C}\right).$$

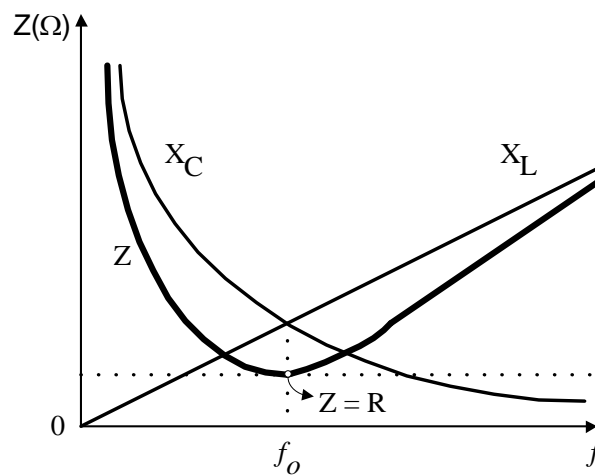


**Figure 5-2:** A series RLC circuit and its impedance phasor diagram.

5. Figure 5-3 illustrates how the impedance of a series RLC circuit changes with frequency. At zero frequency, both  $1/(\omega C)$  and  $Z$  are infinitely large, and  $\omega L$  is zero. You may

recall that at 0 Hz, the capacitor acts like an open circuit, while the inductor acts like a short circuit.

As the frequency increases,  $1/(\omega C)$  decreases, and  $\omega L$  increases. At the frequencies below the resonant frequency  $f_0$ ,  $\omega L < 1/(\omega C)$ , the circuit, whose reactance  $X = \text{Im}\{Z\}$  is negative, is said to be capacitive. At the **resonant frequency**  $f_0$ ,  $\omega L = 1/(\omega C)$ , so the circuit is purely resistive and  $Z = R$ . At the frequencies above  $f_0$ ,  $\omega L > 1/(\omega C)$ , and the circuit, whose reactance  $X = \text{Im}\{Z\}$  is now positive, is said to be inductive. The minimum magnitude of the impedance occurs at a resonant frequency  $f_0$  where  $X_L = X_C$ , but increases in value when the frequency is above or below the resonant point.



**Figure 5-3:** Impedance magnitude of a series RLC circuit as a function of frequency.

6. For the series RLC circuit shown in Figure 5-2, the phasor current through all the circuit elements is given by

$$I = \frac{V_s}{Z} = \frac{V_s}{R + j\left(\omega L - \frac{1}{\omega C}\right)}.$$

The phasor voltages across the resistor, capacitor, and inductor can then be found via

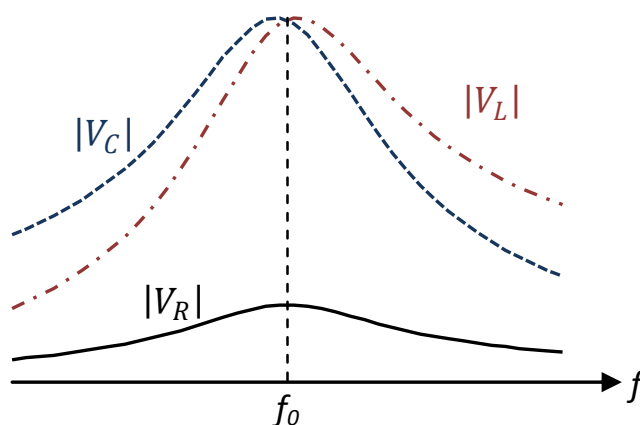
$$V_R = IR, \quad V_C = IZ_C, \quad \text{and} \quad V_L = IZ_L,$$

respectively. Their magnitudes are shown in Figure 5-4-1.

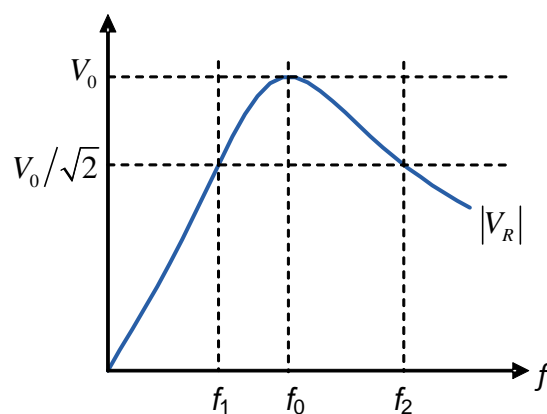
7. Figure 5-4-2 shows the plot of  $|V_R|$ . Three significant points have been marked on the curve. These are  $f_0$ , the **resonant frequency**, and  $f_1$  and  $f_2$ . For series RLC circuits, the  $|V_R|$  is maximum at  $f_0$ . Note that

$$|V_R| = |I|R = \frac{|V_S|R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}},$$

where  $\omega = 2\pi f$ .



**Figure 5-4-1:** Voltage values across the resistor ( $|V_R|$ ), the capacitor ( $|V_C|$ ), and the inductor ( $|V_L|$ ) in a series RLC circuit.



**Figure 5-4-2:** Frequency response of a series RLC circuit.

The frequency  $f$  (or equivalently, the angular frequency  $\omega$ ) that maximizes  $|V_R|$  must be the one that minimizes  $\left(\omega L - 1/(\omega C)\right)^2$ . Because  $\left(\omega L - 1/(\omega C)\right)^2$  is nonnegative, if

we can make the squared term zero then we will get the minimum of its value which in turn will maximize  $|V_R|$ . Therefore, the maximum value of  $|V_R|$  occurs at  $\omega$  such that

$$\omega L = \frac{1}{\omega C},$$

which gives

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ and } f_0 = \frac{1}{2\pi} \frac{1}{\sqrt{LC}}.$$

Points  $f_1$  and  $f_2$  are located at  $1/\sqrt{2}$ , which is approximately 70.7%, of the maximum (at  $f_0$ ) on the curve. They are called the **half power** points, and the (angular) frequency separation between them is called the **bandwidth (BW)** of the circuit. For the series RLC circuit under consideration,

$$BW = 2\pi(f_2 - f_1) = \omega_2 - \omega_1 = \frac{R}{L}.$$

#### Remarks:

- (1) In general, the maximum values for  $|V_L|$  and  $|V_C|$  will **NOT** occur at resonant frequency. (See Figure 5-4-1.)
  - (2) At the resonant frequency, the voltages  $|V_L|$  and  $|V_C|$  across  $L$  and  $C$ , respectively, are equal.
8. For a circuit intended to be frequency selective, the sharpness of the selectivity is a measure of the circuit quality. The quality of the frequency response is described quantitatively in terms of the ratio of the resonance frequency to the bandwidth. This ratio is called the **quality factor (Q)** of the circuit. Therefore,

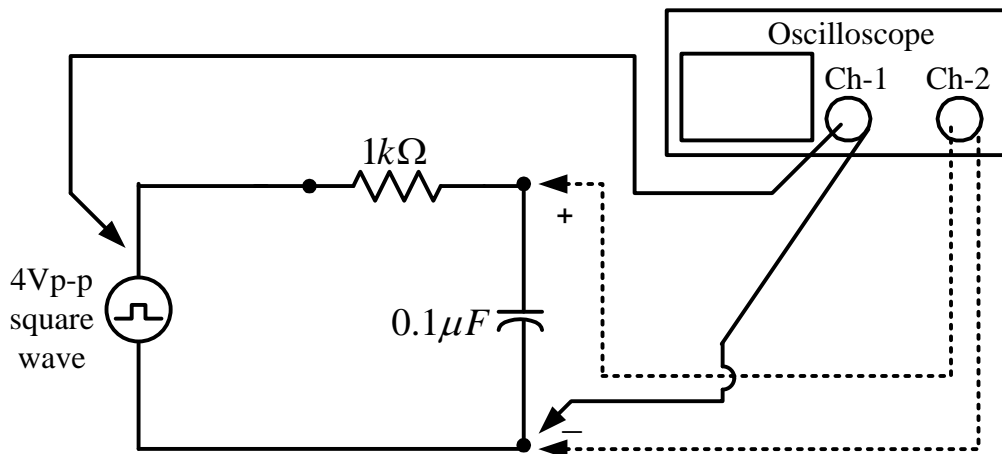
$$Q = \frac{\omega_0}{BW} = \frac{1}{R} \sqrt{\frac{L}{C}}.$$

### III. MATERIALS REQUIRED

- Function generator, multi-meter, and oscilloscope.
- Resistor: 100  $\Omega$ , 1 k $\Omega$ .
- Inductor: 22 mH.
- Capacitors: 0.01  $\mu\text{F}$ , 0.47  $\mu\text{F}$ , and 0.1  $\mu\text{F}$ .

## IV. EXPERIMENTS

### Part A: RC Circuit with Square-Wave Input



**Figure 5-5:** RC Circuit with Voltage Square-Wave Input

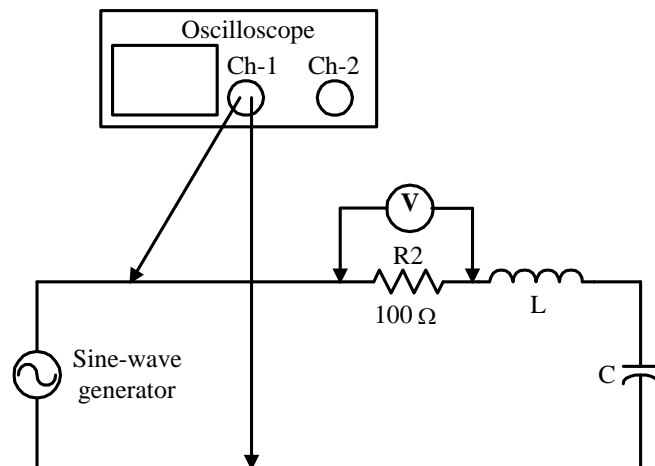
1. Connect the circuit of Figure 5-5, where the oscilloscope is in **DC mode**. Note that Channel 1 is connected to the input voltage while Channel 2 is connected to the output voltage.
2. Set the output of the function generator to be a **square wave** with amplitude of 4 Vp-p and frequency of 500 Hz. Then set the **DC offset level** to 2 V, such that the square-wave waveform has its lower peak at 0 V and its higher peak at 4 V.
3. Adjust volts/div and time/div until the waveforms can be seen on the oscilloscope. Then, draw the voltage waveforms across the function generator (shown by dash line in Figure 5-1), and across the capacitor (shown by solid line) in Table 5-1.
4. Consider the time interval where the capacitor is discharging. Find the time constant  $\tau$  by measuring the time  $t_{0.37}$  which is the amount of time that is needed for the voltage across the capacitor to drop from its maximum value to  $1/e \approx 0.37$  times the maximum value as shown in Figure 5-1-2. Put the value of  $t_{0.37}$  in Table 5-2.
5. Measure the half life  $t_{\text{half}}$  during which the voltage across the capacitor drops to half of the maximum value.
6. Calculate  $\tau$  by the following equation, and record the value in Table 5-2.

$$\tau = \frac{t_{\text{half}}}{\ln 2}$$



- Calculate the product of R and C, and record the value in Table 5-2.

**Part B: Series RLC circuit**



**Figure 5-7:** The series RLC circuit.

- Prepare one 22 mH inductor and three capacitors with values of 0.01, 0.47 and 0.1  $\mu\text{F}$ . Measure all values of the capacitors, and calculate the resonant frequency  $f_o = \omega_o/2\pi = \frac{1}{2\pi\sqrt{LC}}$  Hz for each of the capacitors. Record the calculated resonant frequencies in Table 5-3 under “Calculated resonant frequency”.
- Connect the circuit of Figure 5-7 with  $C = 0.01 \mu\text{F}$ .
- Increase the output of the generator until the scope indicates an 8 V<sub>p-p</sub> voltage. **Maintain/recheck this voltage throughout this part.**
- Observe the rms value of  $V_R$  across the resistor as the frequency of the generator is varied. Record the resonant frequency  $f_o$  at which  $V_R$  is maximum. ~~Measure  $f_o$  and record the value~~ in the “0.01  $\mu\text{F}$ ” row of Table 5-3.  
**Caution:** As you adjust the frequency  $f$ , the voltage across the generator output will change. Readjust it back to 8 V<sub>p-p</sub>.
- Replace the 0.01  $\mu\text{F}$  with a 0.47  $\mu\text{F}$  capacitor. Check the generator output voltage to verify that it is 8 V<sub>p-p</sub>; adjust the voltage if necessary. Find  $f_o$  and record the value in the “0.47  $\mu\text{F}$ ” row of Table 5-3.

6. Replace the  $0.47\mu\text{F}$  with a  $0.1\mu\text{F}$  capacitor. Check the generator output voltage and adjust to maintain  $8\text{ Vp-p}$ . Find  $f_0$  and record the value in the “ $0.1\mu\text{F}$ ” row of Table 5-3.
7. Check the generator output voltage, and adjust the value to maintain at  $8\text{ Vp-p}$ . With  $C = 0.1\mu\text{F}$ , vary and record the frequency according to Table 5-4. Record the corresponding <sup>rms</sup> values of  $V_R$ ,  $V_L$ , and  $V_C$  in Table 5-4, where  $V_L$  and  $V_C$  are the voltage across L and C, respectively.

**Table 5-1. Square wave input voltage and voltage across capacitor**


volts/div = \_\_\_\_\_, time/div = \_\_\_\_\_.

TA Signature:

**Table 5-2. Time constant of an RC circuit**

$\tau$ from $t_{0.37}$	$\tau$ from $t_{\text{half}} / \ln 2$	$\tau$ from RC

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**Table 5-3. Resonant frequency of a series RLC circuit**

Inductor L, mH	Capacitor C, $\mu\text{F}$	Resonant Frequency $f_0$ , Hz	
		Calculated	Measured
22	0.01		
22	0.47		
22	0.1		

TA Signature:

**Table 5-4. Frequency response of a series RLC circuit**

Frequency deviation	Frequency f, Hz	Voltage across R, $V_R$ , V	Voltage across L, $V_L$ , V	Voltage across C, $V_C$ , V
$f_0 - 1000$				
$f_0 - 500$				
$f_0 - 200$				
$f_0 - 100$				
$f_0$				
$f_0 + 100$				
$f_0 + 200$				
$f_0 + 500$				
$f_0 + 1000$				

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## QUESTIONS

### True or False

1. \_\_\_\_\_ In a circuit containing impedance, the current and voltage are 90 degrees out of phase.
2. \_\_\_\_\_ The value of reactance in a series RLC circuit can exceed the magnitude of impedance.
3. \_\_\_\_\_ The value of resistance in a series RLC circuit can exceed the magnitude of impedance.
4. \_\_\_\_\_ In a series RLC circuit, the capacitor rms voltage can be greater than the source rms voltage.
5. \_\_\_\_\_ A series RLC circuit operating above its resonant frequency is inductive.
6. \_\_\_\_\_ There is only one combination of L and C for each resonant frequency.

### Fill in the blanks.

10. The series RLC circuit in Figure 5-2 has an inductance of 100  $\mu\text{H}$ , a capacitance of 200 pF, and a resistance of 10  $\Omega$ . Determine its resonant frequency and bandwidth. Also determine  $X_C$ ,  $X_L$ ,  $I_L$ , and  $V_L$  at 1.0 MHz if the source voltage  $V_S = 1.0 \text{ Vp}$ .

Resonant frequency = \_\_\_\_\_

Bandwidth = \_\_\_\_\_

At 1.0 MHz

$X_C =$  \_\_\_\_\_

$X_L =$  \_\_\_\_\_

$I_L =$  \_\_\_\_\_

$V_L =$  \_\_\_\_\_

11. The series RLC circuit in Figure 5-2 has a capacitance of 25  $\mu\text{F}$ , and a resistance of 10  $\Omega$ . We then adjust the inductance until  $V_R$  is maximum. Determine  $V_R$ ,  $V_L$ , and  $V_C$  at  $\omega = 2000 \text{ rad/sec}$  if the source voltage  $V_S = 50 \text{ Vp}$ .

$V_R =$  \_\_\_\_\_

$V_L =$  \_\_\_\_\_

$V_C =$  \_\_\_\_\_

### Short Answers

12. What are the characteristics of series resonant circuits?
13. Suppose you set the voltage across the output of the signal generator at  $2V_{\text{rms}}$ . You then connect the generator output across a  $100\Omega$  resistor. Now, you measure the voltage across the generator again but you get a value which is significantly less than  $2V_{\text{rms}}$ . Why?
14. How can you to adjust the DC offset of the signal generator?
15. Is changing the DC offset on the signal generator the same as changing the vertical position of the trace in oscilloscope? Explain.
16. What are the differences between the DC and AC mode of the oscilloscope?