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School of Information, Computer and Communication Technology

COURSE : ECS 210 Basic Electrical Engineering Lab

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EXPERIMENT : 03 Network Theorems II: Superposition & Maximum Power Transfer

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I. OBJECTIVES

1. To verify the superposition principle in resistive circuits.
2. To verify the condition of maximum power transfer for a resistive load.

II. BASIC INFORMATION

1. Superposition principle

In any *linear* circuit containing several independent sources, the voltage across or the current flowing through any element can be computed by adding algebraically all the individual voltages or currents caused by each independent source acting alone, with all other independent sources **deactivated**. This is known as **superposition principle**. Steps in applying superposition principle may be summarized as follows.

Step 0: Consider one voltage or current source at a time.

Step 1: Replace the other voltage or current sources with short circuits (zero resistance) or open circuits (infinite resistance), respectively.

Step 2: Determine the particular current or voltage that we want to know as if there were only one source in the circuit—the one which is left activated in Step 1.

Step 3: Take the next source in the circuit, and repeat Steps 1 and 2 for each source.

Step 4: To find the actual current or voltage, add the currents or voltages due to individual sources.

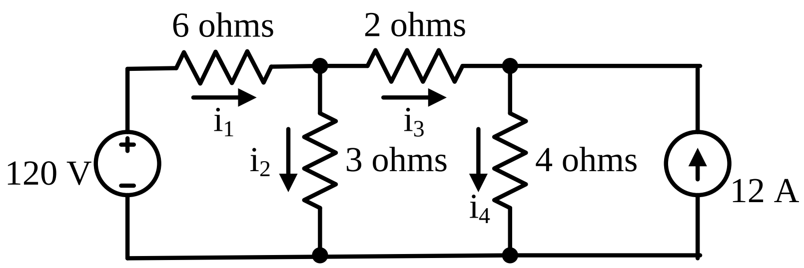

Figure 3-1(a): Circuit to illustrate superposition technique.

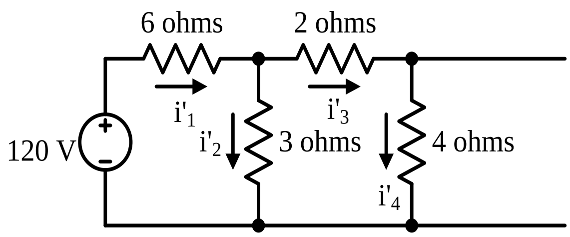

Figure 3-1(b): Circuit in Figure 3-1(a) with the current source deactivated (replaced by an open circuit).

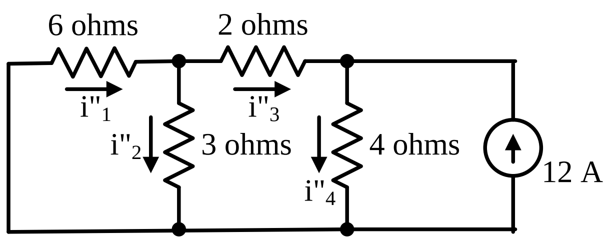

Figure 3-1(c): Circuit in Figure 3-1(a) with the voltage source deactivated (replaced by a short circuit).

From Figure 3-1, the four current values can be determined using superposition principle as follows:

$$i_1 = i'_1 + i''_1$$

$$i_2 = i'_2 + i''_2$$

$$i_3 = i'_3 + i''_3$$

$$i_4 = i'_4 + i''_4$$

2. Maximum power transfer

In many circumstances, it is desirable to obtain the maximum possible power that a given source can deliver to a load. For a *resistive circuit* connected with a load R_L across its two terminals, the condition for maximum power transfer to R_L is that $R_L = R_{TH}$, where R_{TH} is the Thevenin resistance of the circuit with respect to the two terminals. To see why this fact holds, recall that the original resistive circuit can be replaced by its Thevenin equivalent circuit as shown in Figure 3-2. The power p dissipated in R_L can then be expressed as a function of three circuit parameters: V_{TH}, R_{TH}, and R_L:

$$p = i^2 R_L \text{ where } i = \frac{V_{th}}{R_{th} + R_L}.$$

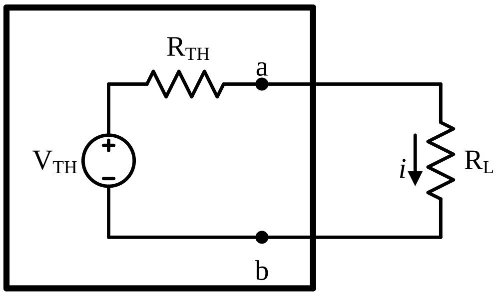

Figure 3-2: A circuit used to determine the value of RL for maximum power transfer.

To find the value of R_L that maximizes the power, elementary calculus says we should solve for the value of R_L when dp/dR_L equals zero. The derivative of p is given by

$$\frac{dp}{dR_L} = 2i\frac{di}{dR_L}R_L + i^2 = 2\frac{V_{th}}{R_{th}+R_L}\left(-\frac{V_{th}}{(R_{th}+R_L)^2}\right) + \left(\frac{V_{th}}{R_{th}+R_L}\right)^2$$
$$= \left(\frac{V_{th}}{R_{th}+R_L}\right)^2\left(-\frac{2R_L}{R_{th}+R_L}+1\right).$$

The value of R_L which makes $\frac{dp}{dR_L} = 0$ is

$$R_L = R_{TH}.$$

Thus, the maximum power transfer occurs when the load resistance R_L equals the Thevenin resistance R_{TH}. As a consequence, the maximum power transferred to R_L equals to

$$p_{\max} = \left(\frac{V_{th}}{R_{th} + R_{th}} \right)^2 R_{th} = \frac{V_{th}^2}{4R_{th}}.$$

II. MATERIALS REQUIRED

- DC power supplies
- Multi-meters
- Resistors (1/4-W):

  820-Ω, 1-kΩ, 1.2-kΩ, 2.2-kΩ, and a potentiometer (variable resistor).

IV. PROCEDURE

Caution: Watch out for the signs of the current and voltage.

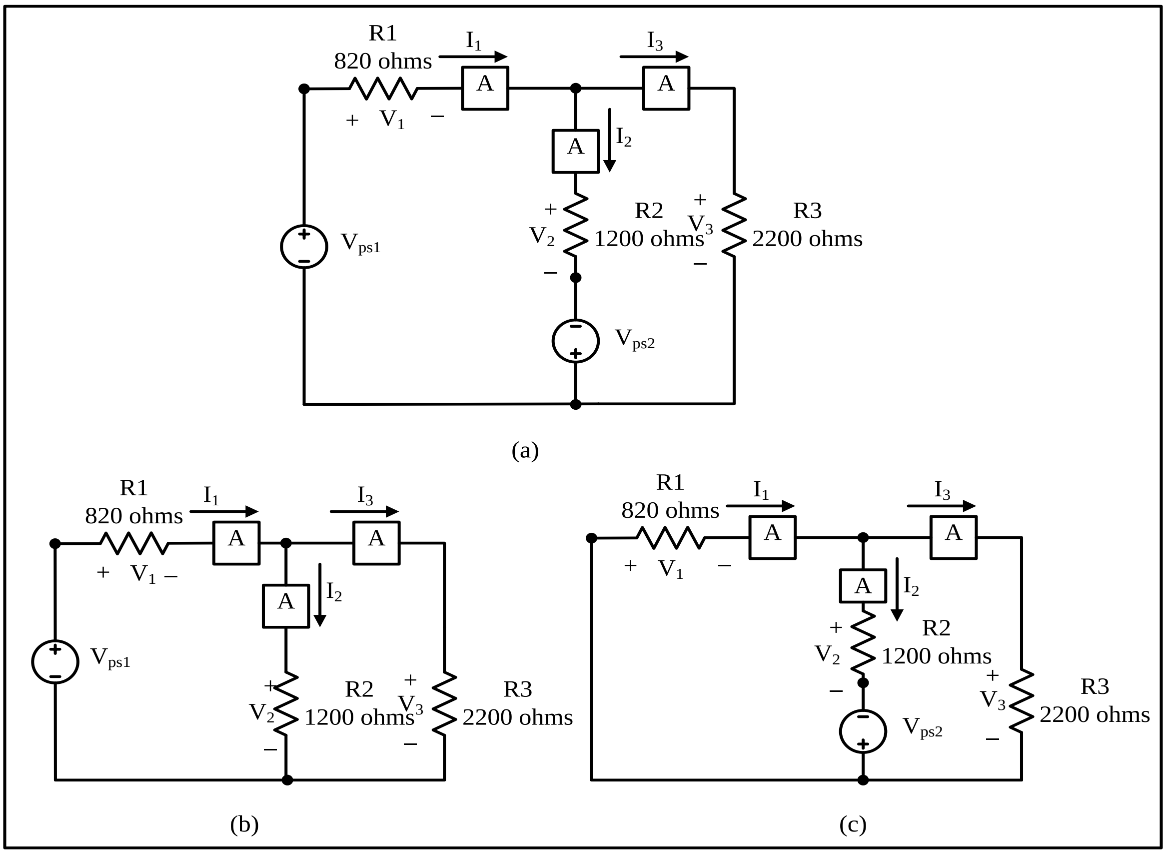

Figure 3-3: Circuit to verify superposition principle: (a) original circuit, (b) modified circuit with Vps1 only, and (c) modified circuit with Vps2 only.

Part A: Superposition principle

1. Figure 3-3(a) shows the circuit under consideration. Let R1= 820 Ω, R2 = 1.2 kΩ, and R3 = 2.2 kΩ. Use a DMM to measure the resistance of each resistor, and record the values in Table 3-1.
2. Turn on the power supply, and adjust its voltage to 15 V. Use this voltage as V_{PS1}, and connect the circuit shown in Figure 3-3(b).
3. Measure the currents I_1, I_2, I_3, and the voltages V_1, V_2, V_3. Record the results in Table 3-1.
4. Adjust ***another*** output of the power supply to 10 V. Use this voltage as V_{PS2}, and connect the circuit in Figure 3-3(c). Measure I_1, I_2, I_3, V_1, V_2, and V_3. Record the results in Table 3-2.
5. With V_{PS1}= 15 V and V_{PS2}= 10 V, connect the circuit given in Figure 3-3(a). Measure I_1, I_2, I_3, V_1, V_2, and V_3, and record the values in Table 3-3.
6. Turn the power supply off. Use the measured values of R1, R2, and R3 with V_{PS1}=15 V and V_{PS2}= 10 V to calculate I_1, I_2, I_3, V_1, V_2, and V_3 by superposition principle. Record the calculated values in Table 3-3, and show the procedure of your analysis in the report.

Part B: Maximum power transfer

1. Let R_C =1 kΩ. Measure the exact value of R_C from the actual resistor and record the value in Table 3-4.
2. Turn on the power supply, and adjust the voltage V_{PS} to 10 V.
3. With a DMM connected directly across R_L (potentiometer), adjust R_L until R_L = 0 Ω. (You will *not* be able to get 0 Ω from the potentiometer. Simply use the lowest value that you can make it be.) Record the exact resistance value in Table 3-4.
4. Connect the circuit in Figure 3-4.

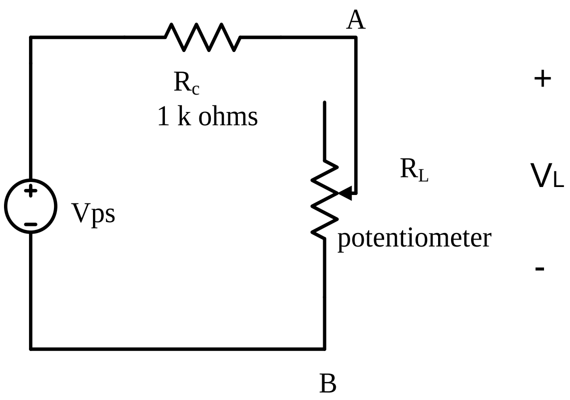

Figure 3-4: A circuit for verifying the maximum power transfer condition.

5. Measure V_L (the voltage across R_L), and record the value in Table 3-4.
6. Disconnect R_L from the circuit.
7. Repeat Steps 3 through 6 for the values of R_L given in Table 3-4.
8. Calculate P_L, the power absorbed by the load R_L, for each value of R_L, and record the result in Table 3-4

   Hint: $P_L = \frac{V_L^2}{R_L}$.

In the report, plot P_L as a function of R_L. Where does the maximum value of P_L occur?

Table 3-1: V_{PS1} = _______________ (A.2) acting alone.

(A.1) Measured resistance: R1= ____________ R2=_____________ R3=_______________

	Current	Voltage
A.3	I_1:	V_1:
	I_2:	V_2:
	I_3:	V_3:

TA Signature:

Table 3-2: V_{PS2} = _______________ (A.4) acting alone.

	Current	Voltage
A.4	I_1:	V_1:
	I_2:	V_2:
	I_3:	V_3:

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Table 3-3: Superposition principle

V_{PS1} = _____________ and V_{PS2} = ____________ acting together.

Measured values (A.5)				Calculated values (A.6)											
				V_{PS1} Only				V_{PS2} Only				V_{PS1} and V_{PS2} together			
Current (mA)		Voltage (V)		Current (mA)		Voltage (V)		Current (mA)		Voltage (V)		Current (mA)		Voltage (V)	
I_1		V_1		I_1		V_1		I_1		V_1		I_1		V_1	
I_2		V_2		I_2		V_2		I_2		V_2		I_2		V_2	
I_3		V_3		I_3		V_3		I_3		V_3		I_3		V_3	

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Table 3-4: Maximum power transfer

R_C = ________________ Ω V_{PS1} = _________ V

$R_L(\Omega)$		$V_L(V)$	Calculated $P_L(mW)$
0			
300			
600			
900			
950			
1000			
1050			
1100			
1400			
1700			
2000			

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QUESTIONS

1. Assume that R_3 is the load of the whole circuit in Figure 3-3(a). Find the value of R_3 for maximum power transfer, where R_1 = 820 Ω and R_2 = 1.2 kΩ as given in the figure. Also determine the maximum power that can be transferred to R_3.

2. If the polarity of the power supply V_{PS2} in Figure 3-3 (a) is reversed, then

   I_1 = ____________ mA V_1 = ____________ V

   I_2 = ____________ mA V_2 = ____________ V

   I_3 = ____________ mA V_3 = ____________ V