ECS203 2015 HW9 Name:______________________________ ID: __________________

Sirindhorn International Institute of Technology

Thammasat University at Rangsit

School of Information, Computer and Communication Technology

ECS 203: Problem Set and Tutorial 11

Semester/Year: **2/2015**
Course Title: Basic Electrical Engineering
Instructor: Asst. Prof. Dr. Prapun Suksompong (prapun@siit.tu.ac.th)
Course Web Site: http://www2.siit.tu.ac.th/prapun/ecs203/

Due date: Not Due

Instructions

1. All phasor should be answered in polar form where the magnitude is positive and the phase is between -180° and 180°.
2. All sinusoid should be answered in the cosine form where the amplitude is positive and the phase is between -180° and 180°.

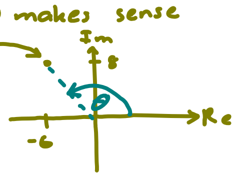
Questions

The first three questions are here to give you a warm-up exercise for the computation that you will encounter throughout chapters 7,8 and 9. You will need to be able to work with complex numbers and many of the calculations will require the use of a calculator.

1) Simplify and then express the following complex numbers in polar form. Make sure that the magnitude values are positive and the phase values are between -180° and 180°.

a) $-6+8j = 10\angle 127^\circ$

Getting an angle $\in (90^\circ, 180^\circ)$ makes sense because $-6+8j$ is here



This can be found by hand via $\sqrt{(-6)^2+8^2} = 10$

b) $\dfrac{50\angle -30^\circ}{10j+5-2j} = \dfrac{50\angle -30^\circ}{5+8j} \approx 5.3\angle -88^\circ$

This can be found by hand via $\dfrac{50}{\sqrt{5^2+8^2}} = \dfrac{50}{\sqrt{89}}$

(Recall that $\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}$.)

2) Simplify and then express the following complex numbers in rectangular form.

a) $-10j+\frac{(3-2j)\times(8+10j)}{(3-2j)+(8+10j)}$ $\approx 3.22 - 11.1\,j$

Alternatively, one can try to work on this part by hand:

$$\frac{(3-2j)\times(8+10j)}{(3-2j)+(8+10j)} = \frac{44+14j}{11+8j} = \frac{(44+14j)(11-8j)}{(11-8j)(11-8j)} = \frac{596-198j}{11^2+8^2} = \frac{596}{185} - \frac{198}{185}j$$

Finally, after adding "$-10j$" to the result, we have

$$\frac{596}{185} - \left(\frac{198}{185}+10\right)j = \frac{596}{185} - \frac{2048}{185}j$$

b) $\left(20\angle-15^\circ\right)\times\frac{100j}{60+100j}$ $\approx 17.15\angle 15.96^\circ$

Alternatively, we can first convert every terms to polar form:

$100j = 100\angle 90^\circ$

$60+100j = 20\sqrt{34}\angle 59^\circ$

Therefore,

$\sqrt{60^2+100^2} = 10\sqrt{36+100} = 20\sqrt{9+25} = 20\sqrt{34}$

$$(20\angle 15^\circ)\times\frac{100j}{60+100j} = \frac{20\times 100}{20\sqrt{34}}\angle\underbrace{(-15^\circ+90^\circ-59^\circ)}_{16^\circ}$$

3) Suppose $\mathbf{V}_S = 20\angle 90^\circ$, $\mathbf{I}_S = 5$, $\mathbf{Z}_1 = -2j$, $\mathbf{Z}_2 = 10j$, $\mathbf{Z}_3 = 8$, $\mathbf{Z}_4 = -2j$, and $\mathbf{Z}_5 = 4$.

Furthermore, suppose

$$\mathbf{I}_3 = \mathbf{I}_S,$$
$$-\mathbf{I}_1\mathbf{Z}_3 - (\mathbf{I}_1-\mathbf{I}_3)\mathbf{Z}_2 - (\mathbf{I}_1-\mathbf{I}_2)\mathbf{Z}_4 = 0, \text{ and}$$
$$-(\mathbf{I}_2-\mathbf{I}_1)\mathbf{Z}_4 - (\mathbf{I}_2-\mathbf{I}_3)\mathbf{Z}_1 - \mathbf{I}_2\mathbf{Z}_5 - \mathbf{V}_S = 0.$$

Find $\mathbf{I}_2$ (in polar form).

We have three unknown variables (phasors) here: $\vec{I}_1$, $\vec{I}_2$, and $\vec{I}_3$. However, because $\vec{I}_3 = \vec{I}_s = 5$, we actually have only two unknowns. So, we will try to reorganize the remaining two equations so that $\vec{I}_1$ and $\vec{I}_2$ are on the LHS and everything else are on the RHS.

$$(-\vec{Z}_3-\vec{Z}_2-\vec{Z}_4)\vec{I}_1 + \vec{Z}_4\vec{I}_2 = -\vec{I}_3\vec{Z}_2$$

$-8-10j-(-2j) = -8-8j$; $\vec{Z}_4 = -2j$; $\vec{I}_3 = 5$, $\vec{Z}_2 = 10j$, $-\vec{I}_3\vec{Z}_2 = -50j$

$(-8-8j)\vec{I}_1 + (-2j)\vec{I}_2 = -50j$

$(+4+4j)\vec{I}_1 + (+j)\vec{I}_2 = +25j \quad (1)$ $\quad \times -\frac{1}{2}$

$$\vec{Z}_4\vec{I}_1 + (-\vec{Z}_4-\vec{Z}_1-\vec{Z}_5)\vec{I}_2 = \vec{V}_s - \vec{I}_3\vec{Z}_1$$

$\vec{Z}_4 = -2j$; $2j+2j-4 = -4+4j$; $20\angle 90^\circ - 5(-2j) = 20j+10j = 30j$

$(-2j)\vec{I}_1 + (-4+4j)\vec{I}_2 = 30j$

$(-j)\vec{I}_1 + (-2+2j)\vec{I}_2 = 15j$ $\quad \times \frac{1}{-j} = j$

$\vec{I}_1 + (-2-2j)\vec{I}_2 = -15$

$\vec{I}_1 = -15 + (2+2j)\vec{I}_2$

Substitute this $\vec{I}_1$ into (1) to get $(4+4j)\left(-15+(2+2j)\vec{I}_2\right) + j\vec{I}_2 = 25j$

which gives $\vec{I}_2 = \frac{25j+15(4+4j)}{(4+4j)(2+2j)+j} = \frac{60+85j}{17j} = 5 - \frac{60}{17}j$

$\approx 5 - 3.53j \approx 6.12\angle -35.2^\circ$

4) [Alexander and Sadiku, 2009, Ex 9.1] Find the amplitude, phase, period, and frequency of the sinusoid

$$v(t) = 12\cos(50t + 10^\circ).$$

Recall that for a sinusoid in standard form

$x(t) = X_m \cos(\omega t + \phi)$

The amplitude is X_m.
The phase is ϕ.
The angular frequency is ω.
The period is $T = \frac{2\pi}{\omega}$.
The frequency is $f = \frac{\omega}{2\pi} = \frac{1}{T}$.

The amplitude is 12.
The phase is 10°.
The angular frequency is $\omega = 50$.
The frequency is $\frac{\omega}{2\pi} = \frac{50}{2\pi} = \frac{25}{\pi} \approx 7.958$.
The period is $\frac{1}{\text{freq.}} = \frac{1}{25/\pi} = \frac{\pi}{25} \approx 0.1257$.

5) Find the phasors (in <u>standard</u> form) corresponding to the following signals.

a) v(t) = 120 sin (10t –50º) V $= 120\cos(10t - 50^\circ - 90^\circ) = 120\cos(10t - 140^\circ)$ (sin → cos)

$\Updownarrow$

$\vec{V} = 120 \angle -140^\circ$ V

b) i(t) = -60cos(30t +10º) mA $= 60\cos(30t + 10^\circ - 180^\circ) = 60\cos(30t - 170^\circ)$ (−cos → cos)

$\Updownarrow$

$\vec{I} = 60 \angle -170^\circ$ mA

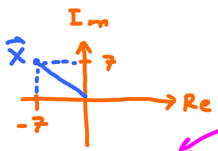
c) i(t) = -8 sin(10t+70º) mA $= 8\cos(10t + 70^\circ - 90^\circ + 180^\circ) = 8\cos(10t + 160^\circ)$ (sin → cos, −cos → cos)

$\Updownarrow$

$\vec{I} = 8 \angle 160^\circ$ mA

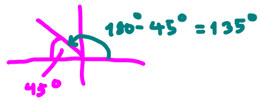
6) [F2010]

a) Find the sinusoid $x(t)$ which is represented by a phasor $\mathbf{X} = -7 + 7j$. Assume $\omega = 100$ rad/s. (Your answer should be a time-dependent sinusoid in standard form.)



we know that the angle in the triangle is 45° because the two sides have the same length.

Therefore, the phase is



Note that you should be able to get this without using a calculator.

$\vec{X} = 7\sqrt{2} \angle 135^\circ \Leftrightarrow x(t) = 7\sqrt{2}\cos(100t + 135^\circ)$ (ω is given.)

Pythagoras' theorem: $\sqrt{7^2 + 7^2} = 7\sqrt{2}$

b) Simplify $x(t) = 7\cos(t - 777^\circ) - 7\sin(t - 77^\circ)$. (Your answer should be a time-dependent sinusoid in standard form.)

$x(t) = 7\cos(t - 777^\circ) - 7\sin(t - 77^\circ)$

$-7\sin(t - 77^\circ) = 7\cos(t - 77^\circ - 90^\circ + 180^\circ) = 7\cos(t + 13^\circ)$

$7\cos(t - 777^\circ) = 7\cos(t - 777^\circ + 720^\circ) = 7\cos(t - 57^\circ)$

Phasor form: $\vec{X} = 7\angle 57^\circ + 7\angle 13^\circ \approx 3.812 - 5.87j + 6.821 + 1.56j$

$= 10.633 - 4.3j \approx 11.47 \angle -22^\circ$

Conversion back to time domain:

$x(t) = 11.47\cos(t - 22^\circ)$

7) [Alexander and Sadiku, 2009, Q9.24a] Find $v(t)$ in the following integrodifferential equation using the phasor approach:

$$v(t)+\int v\,dt = 5\cos\left(t+45^\circ\right).$$

($\omega = 1$)

Recall that
$v(t) \Leftrightarrow \vec{V}$
$\frac{d}{dt}v(t) \Leftrightarrow j\omega\vec{V}$
$\int v(t)dt \Leftrightarrow \frac{\vec{V}}{j\omega}$

Step 1: Conversion to phasor rep.

$$\vec{V} + \frac{\vec{V}}{j\omega} = 5\angle 45^\circ$$

Step 2: Solve for the variable under consideration

$$\vec{V}\left(1+\frac{1}{j}\right) = 5\angle 45^\circ$$

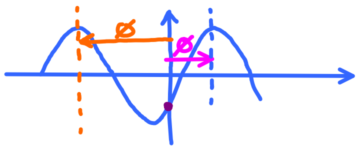
$$\vec{V} = \frac{5\angle 45^\circ}{1-j} = \frac{5\angle 45^\circ}{\sqrt{2}\angle -45^\circ} = \frac{5}{\sqrt{2}}\angle 90^\circ$$

Step 3: Conversion back to time domain

$$v(t) = \frac{5}{\sqrt{2}}\cos\left(t+90^\circ\right) \approx 3.536\cos(t+90^\circ)$$

8) (*) Consider the signal x(t) in Figure 1 below. Suppose $x(0) = -3.356$. Find its phasor.

First we observe that the waveform is the same as cosine function shifted to the right by $|\phi|$ where $|\phi|$ is between 90° and 180°



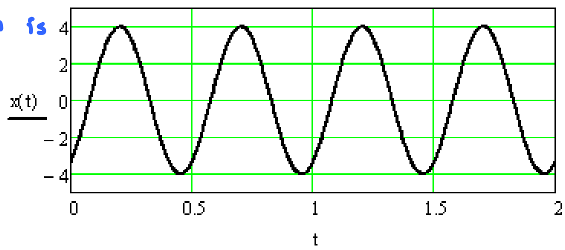


Figure 1

Hint: 1) The amplitude is an integer. Find it first. 2) When t = 0, we also have ωt = 0.

Shifting to the right means ϕ is negative.
So,
$$-180^\circ < \phi < -90^\circ$$

Equivalently, the graph is also the cosine function shifted to the left by ϕ where
$$180^\circ < \phi < 270^\circ$$

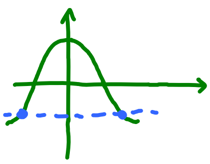
Now,
from the general form of sinusoidal waveform

$$x(t) = A\cos(\omega t+\phi)$$

From the plot, we have $A = 4$.

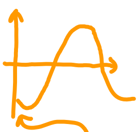
$$x(0) = 4\cos(\phi)$$
$$x(0) = -3.356$$

$$\cos\phi = -\frac{3.356}{4} \approx -0.839$$



Two solutions: ϕ = 147° and −147°

Note that ϕ = 147° will give different graph. Try it! You will get



which start at the wrong position.

Because ϕ must be between −180° and −90°, we know that ϕ = −147°.

Therefore,

$$\vec{X} = 4\angle -147^\circ$$