Sirindhorn International Institute of Technology

Thammasat University at Rangsit

School of Information, Computer and Communication Technology

ECS 203: Problem Set 10

Semester/Year: **2/2015**

Course Title: Basic Electrical Engineering

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Course Web Site: http://www2.siit.tu.ac.th/prapun/ecs203/

Due date: April 18, 5 PM

Instructions

1. Solve all problems. (5 pt)
   a. Write your name and ID on the top of **every** submitted page.
   b. For each part, write your explanation/derivation and answer in the space provided.
2. ONE sub-question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work carefully on all of them.
3. There is no need to submit (or even print out) page 1 (this cover sheet).
4. Late submission will be rejected.
5. ***Write down all the steps*** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Questions

$L = 10 \times 10^{-3}$

1) [Irwin and Nelms, 2015, Ex 6.6] The current in a 10-mH inductor has the waveform shown in Figure 1. Determine the voltage waveform.

Recall $v = L\underbrace{\frac{di}{dt}}_{\text{slope}}$

During these intervals at the beginning/end, slope $= 0$. So, $v = L \times 0 = 0$.

Here, slope $= \frac{20m}{2m} = 10 \Rightarrow v = 10m \times 10 = 100$ [mV]

Here, slope $= \frac{-20m}{2m} = -10 \Rightarrow v = 10m \times (-10) = -100$ [mV]

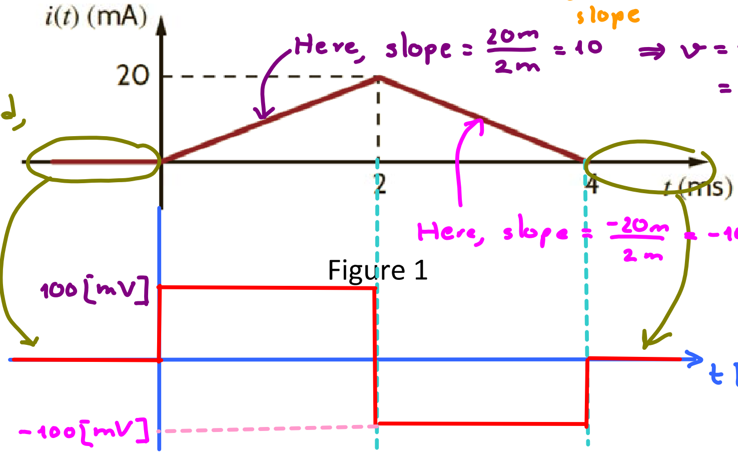
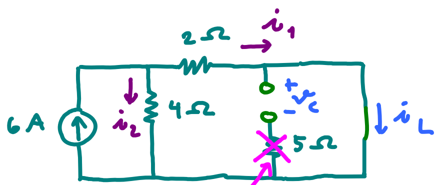


Figure 1

2) [Alexander and Sadiku, 2009, Q6.46] Find v_C, i_L, and the energy stored in the capacitor and inductor in the circuit of Figure 2 under dc, steady-state, conditions.

Recall that, under dc conditions,
capacitor → open circuit
inductor → short circuit



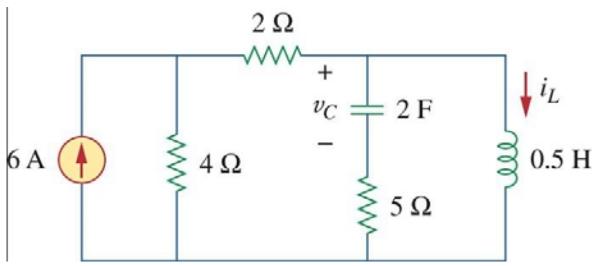
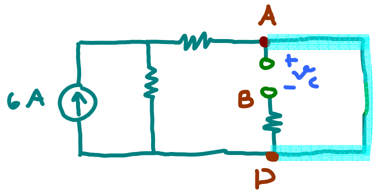


Figure 2



No current through this 5k resistor because the capacitor becomes an open circuit.

Note that there is a short connection from node A to node D; so they are actually the same node.

By current divider formula, $i_1 = 4$ A, $i_2 = 2$ A

$i_L = i_1 = 4$ A

$v_C = v_A - v_B = v_A - v_D = 0$

From the above discussion, because there is no current through the 5k resistor, the Ohm's law implies that there can not be any voltage drop across it.

$\Rightarrow v_B = v_D$

The energy stored in the capacitor is $w_C = \frac{1}{2} C v_C^2 = 0$ J

The energy stored in the inductor is $w_L = \frac{1}{2} L i_L^2 = \frac{1}{2} \times \frac{1}{2} \times 4^2 = 4$ J

$v_C = 0$ V $\quad w_C = 0$ J
$i_L = 4$ A $\quad w_L = 4$ J

3) [Irwin and Nelms, 2015, Ex 6.6] Find the energy stored in the capacitor and inductor in the circuit of Figure 3 under dc, steady-state, conditions.

dc conditions:

capacitor → open circuit

inductor → short circuit

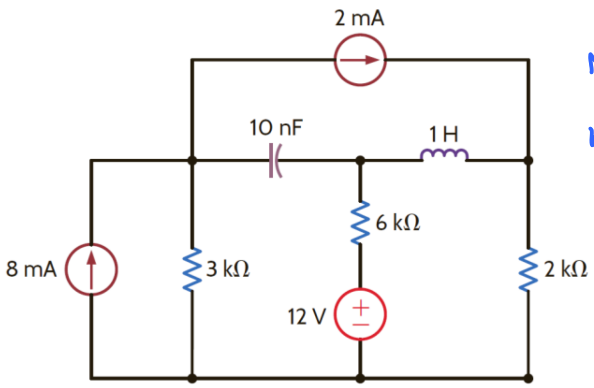


Figure 3

Nodal analysis:

KCL @ node "A"

$$-8m + 2m + \frac{V_A - 0}{3} = 0$$

$$V_A = 18\ mA$$

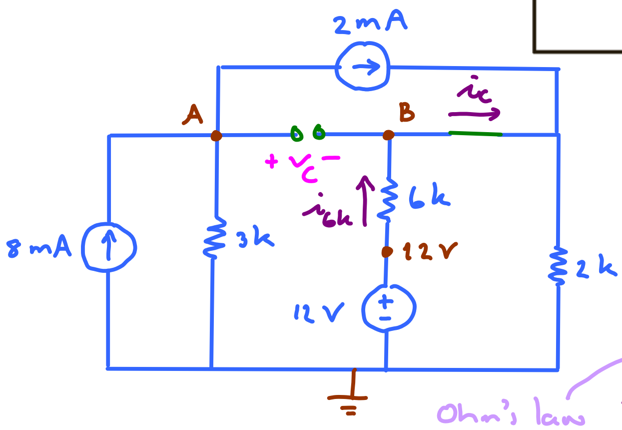
KCL @ node "B"

$$\frac{V_B - 12}{6k} + \frac{V_B}{2k} + (-2m) = 0$$

$$V_B - 12 + 3V_B - 12 = 0$$

$$4V_B = 24$$

$$V_B = 6V$$



$$v_C = v_A - v_B = 12\ V$$

$$w_C = \frac{1}{2} C v_C^2$$

$$= \frac{1}{2} \times 10n \times 12^2 = 720\ nJ = 0.72\ \mu J$$

Ohm's law at the 6kΩ

$$i_L = i_{6k} = \frac{12 - V_B}{6k} = \frac{12-6}{6k} = 1\ mA$$

$$w_L = \frac{1}{2} L i_L^2 = \frac{1}{2} \times 1 \times (1m)^2 = \frac{1}{2}\ \mu J$$

4) [Alexander and Sadiku, 2009, Q6.49] Find the equivalent inductance of the circuit in Figure 4. Assume all inductors are 10 mH.

For this question, although the two terminals for finding equivalent inductance are not explicitly mentioned, we can infer from the given figure that they have to be the terminals a-b here. (Other nodes are not marked as terminals.)

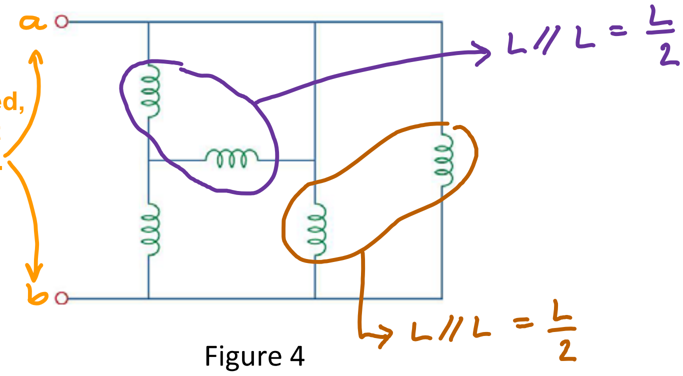
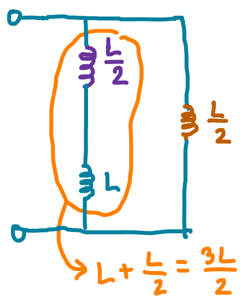


Figure 4

$\equiv$



$$L_{eq} = \frac{3L}{2} // \frac{L}{2} = \frac{L}{2}(3//1) = \frac{L}{2}\,\frac{3}{4} = \frac{3L}{8}$$

When $L = 10\ mH$, $L_{eq} = \frac{3 \times 10}{8} = 3.75\ mH$

5) [Alexander and Sadiku, 2009, Q6.73] Show that the circuit in Figure 5 is a noninverting integrator.

Recall the for ideal op-amp

Rule #1: $i_- = i_+ = 0$

Rule #2: $v_- = v_+ = v$

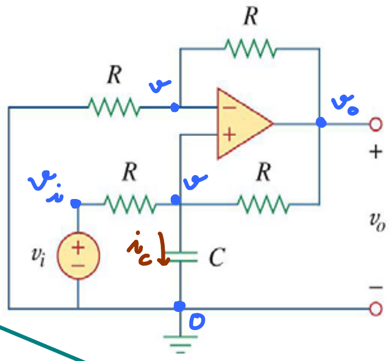


Figure 5

KCL @ "-" input of the op-amp

$$\frac{v-0}{R} + \frac{v-v_o}{R} + 0 = 0 \quad \text{Rule \#1 : } i_- = 0$$

$$2v = v_o \Rightarrow v = \frac{v_o}{2}$$

KCL @ "+" input of the op-amp

$$\frac{v-v_i}{R} + i_c + \frac{v-v_o}{R} + 0 = 0 \quad \text{Rule \#1 : } i_+ = 0$$

$$i_c = C\frac{dv_c}{dt} = C\frac{d}{dt}(v-0) = C\frac{d}{dt}\left(\frac{v_o}{2}\right) = \frac{C}{2}\frac{dv_o}{dt}$$

$$\frac{v-v_i}{R} + \frac{C}{2}\frac{dv_o}{dt} + \frac{v-v_o}{R} + 0 = 0$$

$$\cancel{\frac{v_o}{2}} - v_i + \frac{RC}{2}\frac{dv_o}{dt} + \cancel{\frac{v_o}{2}} - \cancel{v_o} = 0$$

$$\frac{dv_o}{dt} = \frac{2}{RC}v_i$$

integrator

$$v_o(t) = v_o(t_0) + \frac{2}{RC}\int_{t_0}^{t} v_i(\alpha)\,d\alpha$$

no negative sign here → non-inverting