IES 302: Engineering Statistics	**2011/2**

HW 7 — Not Due

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Problem 1. For a normal population with known variance σ^2, answer the following questions:

(a) What is the confidence level for the interval $\overline{x} - 2.14\sigma/\sqrt{n} \leq \mu \leq \overline{x} + 2.14\sigma/\sqrt{n}$?

(b) What is the confidence level for the interval $\overline{x} - 2.49\sigma/\sqrt{n} \leq \mu \leq \overline{x} + 2.49\sigma/\sqrt{n}$?

(c) What is the confidence level for the interval $\overline{x} - 1.85\sigma/\sqrt{n} \leq \mu \leq \overline{x} + 1.85\sigma/\sqrt{n}$?

(d) What value of $z_{\alpha/2}$ gives 98% confidence?

(e) What value of $z_{\alpha/2}$ gives 80% confidence?

(f) What value of $z_{\alpha/2}$ gives 75% confidence?

Problem 2. A confidence interval estimate is desired for the gain in a circuit on a semiconductor device. Assume that gain is normally distributed with standard deviation $\sigma = 20$.

(a) Find a 95% CI for μ when $n = 10$ and $\overline{x} = 1000$.

(b) Find a 95% CI for μ when $n = 25$ and $\overline{x} = 1000$.

(c) Find a 99% CI for μ when $n = 10$ and $\overline{x} = 1000$.

(d) Find a 99% CI for μ when $n = 25$ and $\overline{x} = 1000$.

(e) How does the length of the CIs computed above change with the changes in sample size and confidence level?

(f) How large must n be if the length of the 95% CI is to be 40?

(g) How large must n be if the length of the 99% CI is to be 40?

Problem 3. An article in the *Journal of Agricultural Science* ["The Use of Residual Maximum Likelihood to Model Grain Quality Characteristics of Wheat with Variety, Climatic and Nitrogen Fertilizer Effects" (1997, Vol. 128, pp. 135142)] investigated means of wheat grain crude protein content (CP) and Hagberg falling number (HFN) surveyed in the UK. The analysis used a variety of nitrogen fertilizer applications (kg N/ha), temperature (C), and total monthly rainfall (mm). The data shown below describe temperatures for wheat grown at Harper Adams Agricultural College between 1982 and 1993. The temperatures measured in June were obtained as follows:

$$15.2, 14.2, 14.0, 12.2, 14.4, 12.5, 14.3, 14.2, 13.5, 11.8, 15.2.$$

Assume that the standard deviation is known to be $\sigma = 0.5$.

(a) Construct a 99% two-sided confidence interval on the mean temperature.

(b) Suppose that we wanted the total width of the two-sided confidence interval on mean temperature to be 1.5 degrees Celsius at 95% confidence. What sample size should be used?

Problem 4. Determine the t-percentile $\left(t_{\alpha/2,n-1}\right)$ that is required to construct each of the following two-sided confidence intervals:

(a) Confidence level = 95%, degrees of freedom = 12

(b) Confidence level = 95%, degrees of freedom = 24

(c) Confidence level = 99%, degrees of freedom = 13

(d) Confidence level = 99.9%, degrees of freedom = 15

Problem 5. A research engineer for a tire manufacturer is investigating tire life for a new rubber compound and has built 16 tires and tested them to end-of-life in a road test. The sample mean and standard deviation are 60,139.7 and 3,645.94 kilometers. Assume normality in the population. Find a 95% confidence interval on mean tire life.

Problem 6. An article in the Journal of Composite Materials (December 1989, Vol. 23, p. 1200) describes the effect of delamination on the natural frequency of beams made from composite laminates. Five such delaminated beams were subjected to loads, and the resulting frequencies were as follows (in hertz):

$$230.66, 233.05, 232.58, 229.48, 232.58$$

Assume normality in the population. Calculate a 90% two-sided confidence interval on mean natural frequency.

Problem 7. The heat evolved in calories per gram of a cement mixture is approximately normally distributed. The mean is thought to be 100 and the standard deviation is 2. We wish to test $H_0 : \mu = 100$ versus $H_1 : \mu \neq 100$ with a sample of $n = 9$ specimens.

(a) If the acceptance region is defined as $98.5 \leq \overline{x} \leq 101.5$, find the type I error probability α.

(b) Repeat part (a) using a sample size of $n = 5$ and the same acceptance region.

(c) Find the boundary of the critical region if the type I error probability is

(i) $\alpha = 0.01$ and $n = 9$
(ii) $\alpha = 0.05$ and $n = 9$
(iii) $\alpha = 0.01$ and $n = 5$
(iv) $\alpha = 0.05$ and $n = 5$

(d) Calculate the probability of a type II error if the true mean heat evolved is 103 and

(i) $\alpha = 0.01$ and $n = 9$
(ii) $\alpha = 0.05$ and $n = 9$
(iii) $\alpha = 0.01$ and $n = 5$
(iv) $\alpha = 0.05$ and $n = 5$

(e) From your answers in part (d), describe the relationship among the three quantities: n, α, β.

(f) Calculate the P-value if $n = 9$ and the observed statistic (sample mean) is

(i) $\overline{x} = 98$
(ii) $\overline{x} = 101$
(iii) $\overline{x} = 102$

Problem 8. A manufacturer produces crankshafts for an automobile engine. The wear of the crankshaft after 100,000 miles (0.0001 inch) is of interest because it is likely to have an impact on warranty claims. A random sample of $n = 15$ shafts is tested and $\overline{x} = 2.78$. It is known that $\sigma = 0.9$ and that wear is normally distributed.

(a) Test $H_0 : \mu = 3$ versus $H_1 : \mu \neq 3$ using $\alpha = 0.05$.

(b) What is the P-value for this test?

(c) (Difficult) What is the power of this test if $\mu = 3.25$?

Problem 9. A hypothesis will be used to test that a population mean equals 7 against the alternative that the population mean does not equal 7 with unknown variance. What are the critical values for the test statistic T_0 for the following significance levels and sample sizes?

(a) $\alpha = 0.01$ and $n = 20$

(b) $\alpha = 0.05$ and $n = 12$

(c) $\alpha = 0.10$ and $n = 15$

Problem 10. For the hypothesis test $H_0 : \mu = 7$ against $H_1 : \mu \neq 7$ with variance unknown and $n = 20$, approximate the P-value for each of the following test statistics.

(a) $t_0 = 2.05$

(b) $t_0 = -1.84$

(c) $t_0 = 0.4$

Problem 11. An article in *Growth*: A Journal Devoted to Problems of Normal and Abnormal Growth ["Comparison of Measured and Estimated Fat-Free Weight, Fat, Potassium and Nitrogen of Growing Guinea Pigs" (Vol. 46, No. 4, 1982, pp. 306321)] reported the results of a study that measured the body weight (in grams) for guinea pigs at birth.

$$421.0, 452.6, 456.1, 494.6, 373.8, 90.5, 110.7, 96.4, 81.7, 102.4,$$
$$241.0, 296.0, 317.0, 290.9, 256.5, 447.8, 687.6, 705.7, 879.0, 88.8,$$
$$296.0, 273.0, 268.0, 227.5, 279.3, 258.5, 296.0$$

Test the hypothesis that mean body weight is 300 grams. Use $\alpha = 0.05$.

Problem 12. An article in Concrete Research ["Near Surface Characteristics of Concrete: Intrinsic Permeability" (Vol. 41, 1989)] presented data on compressive strength x and intrinsic permeability y of various concrete mixes and cures. Summary quantities are $n = 14$, $\sum y_i = 572$, $\sum y_i^2 = 23,530$, $\sum x_i = 43$, $\sum x_i^2 = 157.42$, and $\sum x_i y_i = 1697.80$. Assume that the two variables are related according to the simple linear regression model.

(a) Calculate the least squares estimates of the slope and intercept.

(b) Estimate σ^2.

(c) Use the equation of the fitted line to predict what permeability would be observed when the compressive strength is $x = 4.3$.

(d) Give a point estimate of the mean permeability when compressive strength is $x = 3.7$.

(e) Suppose that the observed value of permeability at $x = 3.7$ is $y = 46.1$. Calculate the value of the corresponding residual.