IES 302: Engineering Statistics **2011/2**

HW 6 — March 28

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Instructions

(a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.

(b) It is important that you try to solve all problems. (5 pt)

(c) Late submission will be heavily penalized.

(d) Use second part of Table III Cumulative Standard Normal Distribution from [Montgomery and Runger, 2010] to evaluate the Φ function and all probabilities associated with Gaussian distributions. Hereafter, this table will be referred to as the Φ table.

Problem 1. Let X be a uniform random variable on the interval $[0,1]$. Set

$$A = \left[0, \frac{1}{2}\right), \quad B = \left[0, \frac{1}{4}\right) \cup \left[\frac{1}{2}, \frac{3}{4}\right), \quad \text{and } C = \left[0, \frac{1}{8}\right) \cup \left[\frac{1}{4}, \frac{3}{8}\right) \cup \left[\frac{1}{2}, \frac{5}{8}\right) \cup \left[\frac{3}{4}, \frac{7}{8}\right).$$

Are the events $[X \in A], [X \in B]$, and $[X \in C]$ independent?

Problem 2. Suppose Z is a standard Gaussian random variable.

(a) Use the Φ table to find the following probabilities:

(i) $P[Z < 1.52]$

(ii) $P[Z < -1.52]$

(iii) $P[Z > 1.52]$

(iv) $P[Z > -1.52]$

(v) $P[-1.36 < Z < 1.52]$

(b) Use the Φ table to find the value of c that satisfies each of the following relation.

(i) $P[Z > c] = 0.14$

(ii) $P[-c < Z < c] = 0.95$

Problem 3. The peak temperature T, as measured in degrees Fahrenheit, on a July day in New Jersey is a $\mathcal{N}(85, 100)$ random variable.

Remark: Do not forget that, for our class, the second parameter in $\mathcal{N}(\cdot, \cdot)$ is the variance (not the standard deviation).

(a) Express the cdf of T in terms of the Φ function

Hint: Recall that the cdf of a random variable T is given by $F_T(t) = P[T \leq t]$.

(b) Express each of the following probabilities in terms of the Φ function(s). Make sure that the arguments of the Φ functions are positive. (Positivity is required so that we can use only the second part of Table III Cumulative Standard Normal Distribution from [Montgomery and Runger, 2010] to evaluate the probabilities.)

(i) $P[T > 100]$

(ii) $P[T < 60]$

(iii) $P[70 \leq T \leq 100]$

(c) Evaluate each of the probabilities in part (b) using the Φ table.

Problem 4. In a large class, suppose your instructor tells you that you need to obtain a grade in the top 10% of your class to get an A on a particular exam. From past experience she is able to estimate that the mean and standard deviation on this exam will be 72 and 13, respectively. What will be the minimum grade needed to obtain an A? (Assume that the grades will be approximately normally distributed.)

Problem 5. The incomes of junior executives in a large corporation are approximately normally distributed. A pending cutback will not discharge those junior executives with earnings within \$4900 of the mean. If this represents the middle 80% of the incomes, what is the standard deviation for the salaries of this group of junior executives?

Problem 6. Hours That Children Watch Television: A. C. Neilsen reported that children between the ages of 2 and 5 watch an average of 25 hours of television per week. Assume the variable is normally distributed and the standard deviation is 3 hours. If 20 children between the ages of 2 and 5 are randomly selected, find the probability that the mean of the number of hours they watch television will be greater than 26.3 hours. [Source: Michael D. Shook and Robert L. Shook, The Book of Odds.]

Problem 7. Meat Consumption: The average number of pounds of meat that a person consumes per year is 218.4 pounds. Assume that the standard deviation is 25 pounds and the distribution is approximately normal. [Source: Michael D. Shook and Robert L. Shook, The Book of Odds.]

(a) Find the probability that a person selected at random consumes less than 224 pounds per year.

(b) If a sample of 40 individuals is selected, find the probability that the mean of the sample will be less than 224 pounds per year.

Problem 8. The average age of a vehicle registered in the United States is 8 years, or 96 months. Assume the standard deviation is 16 months. If a random sample of 36 vehicles is selected, find the probability that the mean of their age is between 90 and 100 months. [Source: Harpers Index.]

Problem 9. Consider Bernoulli trials whose success probability for each trial is p. Let X_i be the result ($1 =$ success; $0 =$ failure) of the ith trial. Then, the X_i are Bernoulli random variables. Note that the number of successes in n trials can be found by adding the values of $X_1, X_2, \dots, X_n$. Recall that this number is a binomial(n, p) random variable. Regarding it as a summation of n Bernoulli random variables suggests that it can be approximated by Gaussian random variable with the same expected value and variance. Use this approximation to solve the following problem:

A student has passed a final exam by supplying correct answers for 26 out of 50 multiple-choice questions. For each question, there was a choice of three possible answers, of which only one was correct. The student claims not to have learned anything in the course and not to have studied for the exam, and says that his correct answers are the product of guesswork. Use the Φ table to determine whether you should believe him.

Hint:

(a) Calculate the probability of identifying 26 or more correct answers through guesswork. If this probability is small, you judge that the student is bluffing.

(b) Recall that the expected value and variance of Binomial(n, p) random variable are np and $np(1-p)$, respectively.