Problem 1. The input X and output Y of a system subject to random perturbations are described probabilistically by the following joint pmf matrix:

$X \backslash Y$	2	4	6
3	0.1	0.1	0
4	0.2	0.3	0
7	0	0	0.3

Find the following quantities:

(a) the marginal pmf $p_X(x)$

(b) the marginal pmf $p_Y(y)$

(c) $\mathbb{E}X$

(d) $\operatorname{Var} X$

(e) $\mathbb{E}Y$

(f) $\operatorname{Var} Y$

(g) $\mathbb{E}[XY]$

(h) $\mathbb{E}[(X - Y^3)(X + 1)]$

(i) $\operatorname{Cov}[X, Y]$

Problem 2. Suppose you make 13 pairs of observations:

x	y
1	14.1338
4	32.3236
1	5.9754
5	35.785
7	52.4688
6	49.5751
10	77.6489
2	13.5761
6	49.7059
3	28.5717
7	51.8538
3	27.0028
4	27.4189

Let's denote these pairs of values by $(x_1, y_1), (x_2, y_2), \ldots, (x_{13}, y_{13})$. You want to report the relationship between the x and the y by a linear (affine) expression $y = ax + b$. Find the values of a and b that minimize

$$\sum_{k=1}^{13} (y_k - ax_k - b)^2.$$