Sirindhorn International Institute of Technology

Thammasat University at Rangsit

School of Information, Computer and Communication Technology

ET601: Problem Set 6

Due date: February 27, 2013 (Thursday), 1 PM

Instructions

a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.
b) It is important that you try to solve all problems. (5 pt)
c) Submit your work as one pdf file (which contains the solution for all the questions). The PDF file name should be ET601_HW6_FIRSTNAME.pdf in which the FIRSTNAME part is replaced by your first name.
d) The solution should contain both the MATLAB codes and the resulting figures. If answers are also displayed in the command window, they should be captured and shown in your solution as well.
e) Late submission will be heavily penalized.

1. Consider the Multiplicative Congruential Generator (MCG) with the recursion $x_n = ax_{n-1} \bmod m$. We have seen in class that when a = 3 and m = 7, the resulting sequence has maximal length (of the cycle).
   Here, we want to explore this property when the value of *a* is changed. We will consider a = 1, 2, 3,… , 100.
   Write a MATLAB script to answer the following questions.
   a. How many of the *a* above give maximal length sequence?
   b. Among the hundred *a* values that we test, how many are prime numbers?
      Hint: The function `isprime` may be useful here.
   c. Among the *a* values that give maximal length sequence, how many are prime numbers?

2. In class we have seen the script which creates n observations (realizations) sampled from the pmf

$$p_X(x) = \begin{cases} 1/6, & x = 3, \\ 1/3, & x = 4, \\ 1/2, & x = 8, \\ 0, & \text{otherwise.} \end{cases}$$

The script is shown in the box below.

   a. What is the relationship between the `dum` variable and `hist(X,S_X)` a few lines below?
   b. Replace the expression "`hist(X,S_X)/n;`" in the code by another expression which utilize the `dum` variable instead of `hist(X,S_X)`.

```
Clear all; close all;

S_X = [3 4 8];
p_X = [1/6 1/3 1/2];

n = 1e6;

F_X = cumsum(p_X);
U = rand(1,n);
[dum,V] = histc(U,[0 F_X/F_X(end)]);
X = S_X(V);

rf = hist(X,S_X)/n;
stem(S_X,rf,'rx')
hold on
stem(S_X,p_X,'bo')
xlim([min(S_X)-1,max(S_X)+1])
legend('Rel. freq. from sim.'...
    ,'pmf p_X(x)')
xlabel('x')
grid on
```

3. In class we have seen the script GenRV_Geo_while.m which creates n observations (realizations) sampled from the geometric$_1$ pmf with parameter p. The script is shown in the table below. Instead of using the `histc` command to find the bin index that the uniformly distributed on (0,1) falls into, this implementation is based on the use of `while` loop to successively construct the boundaries of the bins. The script also verifies the generated values by comparing the relative frequencies with the theoretical pmf.

   Modify the script so that it creates n observations (realizations) sampled from the Poisson pmf with parameter α.

```
p = 1/3; n = 1e6;
U = rand(1,n);
V = zeros(size(U)); %Preallocation
for i = 1:n % Consider individual element in the U vector
    k = 1; % The first bin. k is the bin index
```

```
    pk = p; F = p; % Boundary of the first bin
    while (U(i) > F) % Check whether U(i) is beyond
                     % the current bin
        k = k+1; % Consider the next bin
        pk = pk*(1-p); % Width of the next bin
        F = F + pk; % Boundary of the next bin
    end
    V(i) = k;
end
X = V;
```

```
X_Range = 1:10; % Only plots the first 10 values
p_X = p*(1-p).^(X_Range-1); % The theoretical pmf
rf = hist(X,X_Range)/n;
stem(X_Range,rf,'rx')
hold on
stem(X_Range,p_X,'bo')
xlim([min(X_Range)-1,max(X_Range)+1])
legend('Rel. freq. from sim.'...
    ,'pmf p_X(x)')
xlabel('x')
grid on
```

The figure below show the expected resulting plots when $n = 10^6$ and $\alpha = 2$.

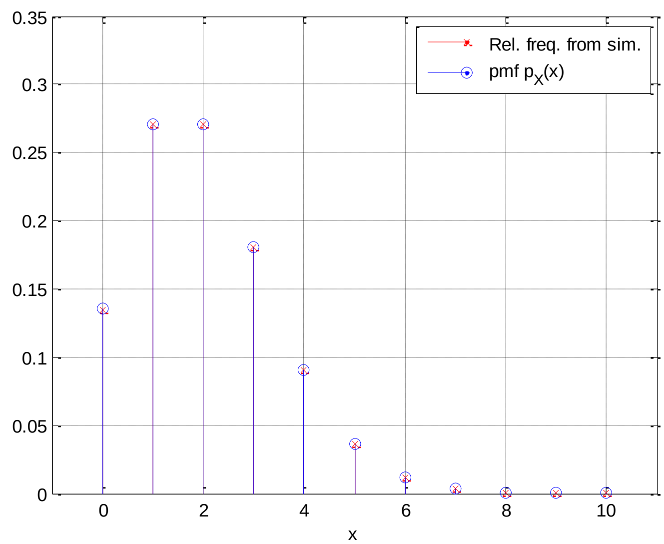

Figure 1