ET 601: Computer Applications for Engineers **2013/2**

HW Solution 5 — Due: January 29

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Instructions

(a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.

(b) It is important that you try to solve all problems. (5 pt)

(c) Submit your work as one pdf file (which contains the solution for all the questions). The PDF file name should be ET601_HW5_FIRSTNAME.pdf in which the FIRSTNAME part is replaced by your first name.

(d) For question that involved MATLAB, the solution should contain both the MATLAB codes and the resulting figures. If answers are also displayed in the command window, they should be captured and shown in your solution as well.

(e) Late submission will be heavily penalized.

Problem 1. The Benford pmf is given by

$$p_X(x) = \begin{cases} \log_{10}\left(1+\frac{1}{d}\right), & x = 1,2,3,\dots 9, \\ 0, & \text{otherwise.} \end{cases}$$

This distribution describes the the relative frequency of the first (leading) digit in many (but not all) real-life sources of data.

(a) Write a MATLAB script to plot the Benford pmf.

(b) In MATLAB, there are many built-in data sources that we can use. Plot the relative frequency of the first (leading) digit in each of these sources. Check whether they agree with the Benford pmf.

  (i) The data saved in the vector `n` in the built-in file `quake.mat`. (This file contains 200Hz data from the October 17, 1989 Loma Prieta earthquake in the Santa Cruz Mountains. In the workspace now are three variables containing time traces from an accelerometer in the Natural Sciences' building at UC Santa Cruz. The accelerometer recorded the main shock of the earthquake. The variables `n`, `e`, `v` refer to the three directional components measured by the instrument, which was aligned parallel to the fault, with its N direction pointing in the direction of Sacramento.)

(ii) The data saved in the vector `y` in the built-in file `laughter.mat`.

(c) In economics, there is evidence that the income is distributed approximately log-normally. It is known that if a random variable Y is normal, then $X = e^Y$ has a log-normal distribution. In this part, generate (many realizations of) Y by the `randn` command. Then, plot the relative frequency of the first (leading) digit of the corresponding values of $X = e^Y$ to check whether they agree with the Benford pmf.

(d) Find your own data set that conform to Benford pmf. The data must be mainly related to your country/place of birth. The data should not be taken from a source that has already been checked with the Benford pmf before.

Problem 2. Write a MATLAB script which uses MATLAB's Symbolic Math Toolbox to verify the following identities.

(a) $\sum_{k=1}^{m} k^3 = \frac{m^4}{4} + \frac{m^3}{2} + \frac{m^2}{4}$

(b) $\int_0^\infty x^3 e^{-ax^2}\, dx = \frac{1}{2a^2}$ when $a > 0$

(c) $\int_0^\infty \frac{x^3}{e^x - 1}\, dx = \frac{\pi^4}{15}$

Problem 3. Write a MATLAB script which uses MATLAB's Symbolic Math Toolbox and the formulas provided in class to find the expected value and variance of the following random variables

(a) A ***geometric*** random variable X whose pmf is given by

$$p_X(x) = \begin{cases} p(1-p)^{x-1}, & x = 1, 2, \ldots \\ 0, & \text{otherwise.} \end{cases}$$

where $0 < p < 1$.

(b) A ***Poisson*** random variable X whose pmf is given by

$$p_X(x) = \begin{cases} e^{-\alpha} \frac{\alpha^x}{x!}, & x = 0, 1, 2, \ldots \\ 0, & \text{otherwise} \end{cases}$$

where $\alpha > 0$.

(c) An ***exponential*** random variable X whose pdf is given by

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0, \\ 0, & x \leq 0 \end{cases}$$

where $\lambda > 0$.

(d) A ***Gaussian*** random variable X whose pdf is given by

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}$$

where $\sigma > 0$.